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WEIGHTED COMPOSITION OPERATORS ACTING BETWEEN WEIGHTED BERGMAN SPACES AND WEIGHTED BANACH SPACES OF HOLOMORPHIC FUNCTIONS ON THE UNIT BALL

Elke Wolf

Abstract. We characterize boundedness and compactness of weighted composition operators acting between weighted Bergman spaces $A_{v,p}$ and weighted Banach spaces H_w^∞ of holomorphic functions on the open unit ball of C^N, $N \geq 1$. Moreover, we give a sufficient condition for such an operator acting between weighted Bergman spaces $A_{v,p}$ and $A_{w,p}$ on the unit ball to be bounded.

1. Introduction

Let v and w be strictly positive continuous and bounded functions (*weights*) on the open unit ball B_N of $\mathbf{C}^N$. Moreover, let $H(B_N)$ denote the class of all analytic functions on B_N. Analytic self-maps $\phi : B_N \to B_N$ and functions $\psi \in H(B_N)$ induce weighted composition operators $\psi C_\phi \colon H(B_N) \to H(B_N), f \mapsto \psi(f \circ \phi)$.

We are interested in weighted composition operators acting on weighted Bergman spaces

$$A_{v,p} := \left\{ f \in H(B_N)\ ;\ \|f\|_{v,p} := \left(\int_{B_N} |f(z)|^p v(z)\, dV(z) \right)^{\frac{1}{p}} < \infty \right\},$$

where $dV(z)$ is the normalized Lebesgue measure such that $V(B_N) = 1$ and $1 \leq p < \infty$. Furthermore, we study weighted composition operators from weighted Bergman spaces $A_{v,p}$ to weighted Banach spaces of holomorphic functions

$$H_w^\infty := \left\{ f \in H(B_N)\ ;\ \|f\|_w := \sup_{z \in B_N} w(z)|f(z)| < \infty \right\}.$$

Recently, the subject of composition operators and weighted composition operators acting on various spaces of analytic functions has been of great interest, see e.g.

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[2–8], [10–12], [15–20], [22–23]. This list of articles is only an assortment of papers on this topic.

In [23] we considered weighted composition operators between weighted Bergman spaces and weighted Banach spaces of holomorphic functions on the unit disk where the involved weights belonged to a special class. This result was generalized by S. Stević to the unit ball (see [22]).

In this article we study the boundedness of a weighted composition operator acting between weighted Bergman spaces on the unit ball and characterize boundedness and compactness of weighted composition operators between weighted Bergman spaces and weighted Banach spaces of holomorphic functions on the unit ball for a different class of weights than the one used in [23] or [22].

2. Preliminaries

For notation and detailed information on (weighted) composition operators we refer the reader to the excellent monographs [9] and [21].

We need some geometric facts about the unit ball. Fix a point $\alpha \in B_N$ and let P_α be the orthogonal projection of $\mathbf{C}^N$ onto the space

$$[\alpha] = \{\lambda\alpha\ ;\ \lambda \in \mathbf{C}\}$$

generated by α. Thus $P_0(z) = 0$ and $P_\alpha(z) = \frac{\langle z,\alpha\rangle}{\langle \alpha,\alpha\rangle}\alpha$, $\alpha \neq 0$, where $\langle z,p\rangle = \sum_{k=1}^N z_k\overline{p}_k$ for every $z = (z_1,\dots,z_N), p = (p_1,\dots,p_N) \in \mathbf{C}^N$. Let $Q_\alpha(z) = z - P_\alpha(z)$ be the projection onto the orthogonal complement of $[\alpha]$ and let $s_\alpha = (1 - |\alpha|^2)^{\frac{1}{2}}$. Now define

$$\varphi_\alpha(z) := \frac{\alpha - P_\alpha(z) - s_\alpha Q_\alpha(z)}{1 - \langle z,\alpha\rangle} \text{ and}$$

$$\sigma_\alpha(z) := \frac{(1-|\alpha|^2)}{(1-\langle z,\alpha\rangle)^2} \quad \text{for every } z \in B_N.$$

In the sequel we consider weights of the following type. Let ν be a holomorphic function on the open unit disk D of the complex plane, non-vanishing, strictly positive on $[0,1)$ and such that $\lim_{r\to 1}\nu(r) = 0$. Then we define the weight v by

$$v(z) := \nu(\langle z,z\rangle) = \nu(|z|^2) \text{ for every } z \in B_N.$$

Next, we give some illustrating examples of weights of this type:

(i) Consider $\nu(z) = (1-z)^\alpha$, $\alpha \geq 1$. Then the corresponding weight is the so-called standard weight $v(z) = (1-|z|^2)^\alpha$.

(ii) Select $\nu(z) = e^{-\frac{1}{(1-z)^\alpha}}$, $\alpha \geq 1$. Then we obtain the weight $v(z) = e^{-\frac{1}{(1-|z|^2)^\alpha}}$.

(iii) Choose $\nu(z) = \sin(1-z)$ and the corresponding weight is given by $v(z) = \sin(1-|z|^2)$.

(iv) Put $\nu(z) := \frac{1}{1-\log(1-z)}$. Then we get the weight $v(z) = \frac{1}{1-\log(1-|z|^2)}$.

For the fixed point $\alpha \in B_N$ we introduce a function

$$v_\alpha(z) := \nu(\langle z, \alpha\rangle) \text{ for every } z \in B_N.$$

Since ν is a holomorphic function on D, v_α is holomorphic on B_N. Later on, we will need the extra assumption $\sup_{\alpha\in B_N} \sup_{z\in B_N} \frac{v(z)}{|v_\alpha(z)|} \leq M < \infty$. Obviously, the weight given in (ii) does not satisfy this condition, but for the weight in (i) we obtain

$$\frac{v(z)}{|v_\alpha(z)|} = \frac{1-|z|^2}{|1-\langle z,\alpha\rangle|} \leq \frac{1-|z|^2}{1-|z|} \leq 1+|z| \leq 2$$

for every $z \in B_N$. Thus, the standard weights satisfy the extra assumption. Moreover, we have that

$$|1-\log(1-\langle z,\alpha\rangle)| \leq 1-\log(1-|z|)$$

for every $z \in B_N$ and the function $\frac{1-\log(1-|z|)}{1-\log(1-|z|^2)}$ is continuous and tends to 1 if $|z| \to 1$. Hence the weight in (iv) fulfils the assumption. Similar calculations show that (iii) also satisfies the additional condition.

3. Boundedness of operators $\psi C_\phi : A_{v,p} \to A_{w,p}$

We first need the following auxiliary result, which generalizes the result in [24].

Lemma 1. *Let v be a weight as defined in the previous section such that $\sup_{z\in B_N} \sup_{a\in B_N} \frac{v(z)|v_a(\varphi_a(z))|}{v(\varphi_a(z))} \leq C < \infty$. Then*

$$|f(z)| \leq \frac{C^{\frac{1}{p}}}{\left(\int_{B_N} v(t)\, dV(t)\right)^{\frac{1}{p}} (1-|z|^2)^{\frac{N+1}{p}} v(z)^{\frac{1}{p}}} \|f\|_{v,p}$$

for every $z \in B_N$ and every $f \in A_{v,p}$.

Proof. Let $\alpha \in B_N$ be an arbitrary point. Consider the map

$$T_\alpha : A_{v,p} \to A_{v,p},\ T_\alpha(f(z)) = f(\varphi_\alpha(z))\sigma_\alpha(z)^{\frac{N+1}{p}} v_\alpha(\varphi_\alpha(z))^{\frac{1}{p}}.$$

Then a change of variables gives

$$\begin{aligned}
\|T_\alpha f\|_{v,p}^p &= \int_{B_N} v(z)|f(\varphi_\alpha(z))|^p |\sigma_\alpha(z)|^{N+1} |v_\alpha(\varphi_\alpha(z))|\, dV(z) \\
&= \int_{B_N} \frac{v(z)|v_\alpha(\varphi_\alpha(z))|}{v(\varphi_\alpha(z))} |f(\varphi_\alpha(z))|^p |\sigma_\alpha(z)|^{N+1} v(\varphi_\alpha(z))\, dV(z) \\
&\leq \sup_{z\in B_N} \frac{v(z)|v_\alpha(\varphi_\alpha(z))|}{v(\varphi_\alpha(z))} \int_{B_N} |f(\varphi_\alpha(z))|^p |\sigma_\alpha(z)|^{N+1} v(\varphi_\alpha(z))\, dV(z) \\
&\leq C \int_{B_N} v(t)|f(t)|^p\, dV(t) = C\|f\|_{v,p}^p.
\end{aligned}$$

The mean value property for the non-weighted Bergman space A_p says that for $f \in A_p$ we have that

$$|f(0)|^p = \int_{B_N} |f(z)|^p \, dV(z) = \frac{1}{\int_{B_N} dV(z)} \int_{B_N} |f(z)|^p \, dV(z).$$

In our setting we consider a new measure $d\mu(z) := v(z)\, dV(z)$ and, since the weight v is radial, obtain for functions $h \in A_{v,p}$ the following equality

$$\begin{aligned} |h(0)|^p &= \frac{1}{\int_{B_N} d\mu(z)} \int_{B_N} |h(z)|^p \, d\mu(z) \\ &= \frac{1}{\int_{B_N} v(z)\, dV(z)} \int_{B_N} |h(z)|^p v(z)\, dV(z) \end{aligned}$$

Now put $g(z) := (T_\alpha f)(z)$ for every $z \in B_N$. By the arguments we used above

$$|g(0)|^p \int_{B_N} v(z)\, dV(z) \le C\|f\|_{v,p}^p.$$

Hence

$$|g(0)|^p \int_{B_N} v(z)\, dV(z) = |f(\alpha)|^p (1-|\alpha|^2)^{N+1} v(\alpha) \int_{B_N} v(z)\, dV(z) \le C\|f\|_{v,p}^p.$$

Thus $|f(\alpha)| \le \frac{C^{\frac{1}{p}} \|f\|_{v,p}}{\left(\int_{B_N} v(t)\, dV(t)\right)^{\frac{1}{p}} (1-|\alpha|^2)^{\frac{N+1}{p}} v(\alpha)^{\frac{1}{p}}}$. Since α was arbitrary, the claim follows. ∎

Now, we can give the following sufficient condition for the boundedness of an operator $\psi C_\phi : A_{v,p} \to A_{w,p}$.

Proposition 2. *Let w be a weight and v be a weight as in Lemma 1. If*

$$\sup_{z \in B_N} \frac{|\psi(z)| w(z)^{\frac{1}{p}}}{(1-|\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} < \infty,$$

then the operator $\psi C_\phi : A_{v,p} \to A_{w,p}$ is bounded.

Proof. Applying Lemma 1 we get for $f \in A_{v,p}$

$$\begin{aligned} \|\psi C_\phi f\|_{w,p}^p &= \int_{B_N} |\psi(z)|^p |f(\phi(z))|^p w(z)\, dV(z) \\ &\le \int_{B_N} \frac{C|\psi(z)|^p}{\left(\int_{B_N} v(t)\, dV(t)\right)(1-|\phi(z)|^2)^{N+1} v(\phi(z))} w(z) \|f\|_{v,p}^p \, dV(z) \\ &\le \sup_{z \in B_N} \frac{C w(z) |\psi(z)|^p}{\left(\int_{B_N} v(t)\, dV(t)\right)(1-|\phi(z)|^2)^{N+1} v(\phi(z))} \|f\|_{v,p}^p \end{aligned}$$

and the claim follows. ∎

4. Boundedness and compactness of operators $\psi C_\phi : A_{v,p} \to H_w^\infty$

Next, we turn our attention to the setting of weighted composition operators acting between weighted Bergman spaces and weighted Banach spaces of holomorphic functions.

THEOREM 3. *Let w be a weight and v be a weight as in Lemma 1 such that there is $M > 0$ with $\sup_{\alpha \in B_N} \sup_{z \in B_N} \frac{v(z)}{|v_\alpha(z)|} \le M$. Then the weighted composition operator $\psi C_\phi : A_{v,p} \to H_w^\infty$ is bounded if and only if*

$$\sup_{z \in B_N} \frac{w(z)|\psi(z)|}{(1-|\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} < \infty.$$

Proof. First, suppose that $\sup_{z \in B_N} \frac{w(z)|\psi(z)|}{(1-|\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} < \infty$. By Lemma 1 we have

$$|f(z)| \le \frac{C^{\frac{1}{p}} \|f\|_{v,p}}{\left(\int_{B_N} v(t)\, dV(t)\right)^{\frac{1}{p}} (1-|z|^2)^{\frac{N+1}{p}} v(z)^{\frac{1}{p}}}$$

for all $z \in B_N$ and $f \in A_{v,p}$. Thus, for $f \in A_{v,p}$ we get

$$\begin{aligned}\|\psi C_\phi f\|_w &= \sup_{z \in B_N} w(z)|\psi(z)||f(\phi(z))| \\ &\le \sup_{z \in B_N} \frac{C^{\frac{1}{p}} w(z)|\psi(z)|}{\left(\int_{B_N} v(t)\, dV(t)\right)^{\frac{1}{p}} v(\phi(z))^{\frac{1}{p}} (1-|\phi(z)|^2)^{\frac{N+1}{p}}} \|f\|_{v,p}.\end{aligned}$$

For the converse let $\alpha \in B_N$ be fixed. Choose now $f_\alpha(z) := \frac{\sigma_{\phi(\alpha)}(z)^{\frac{N+1}{p}}}{v_{\phi(\alpha)}(z)^{\frac{1}{p}}}$ for every $z \in B_N$. Then a change of variables yields

$$\begin{aligned}\|f_\alpha\|_{v,p}^p &= \int_{B_N} |f_\alpha(z)|^p v(z)\, dV(z) = \int_{B_N} \frac{1}{|v_{\phi(\alpha)}(z)|} |\sigma_{\phi(\alpha)}(z)|^{N+1} v(z)\, dV(z) \\ &\le \sup_{\alpha \in B_N} \sup_{z \in B_N} \frac{v(z)}{|v_{\phi(\alpha)}(z)|} \int_{B_N} |\sigma_{\phi(\alpha)}(z)|^{N+1}\, dV(z) \\ &\le M \int_{B_N} |\sigma_{\phi(\alpha)}(z)|^{N+1}\, dV(z) = M \int_{B_N} dV(t) = M.\end{aligned}$$

Next, we assume that there is a sequence $(z_n)_{n \in \mathbf{N}} \subset B_N$ such that $|\phi(z_n)| \to 1$ and

$$\frac{|\psi(z_n)|w(z_n)}{v(\phi(z_n))^{\frac{1}{p}} (1-|\phi(z_n)|^2)^{\frac{N+1}{p}}} \ge n$$

for every $n \in \mathbf{N}$. Thus consider now $f_n(z) := \frac{\sigma_{\phi(z_n)}(z)^{\frac{N+1}{p}}}{v_{\phi(z_n)}(z)^{\frac{1}{p}}}$ for every $n \in \mathbf{N}$ as defined above. Then we obtain that $(f_n)_n$ is a bounded sequence in $A_{v,p}$ and thus we can find a constant $c > 0$ such that

$$c \ge w(z_n)|\psi(z_n)||f_n(\phi(z_n))| = \frac{w(z_n)|\psi(z_n)|}{v(\phi(z_n))^{\frac{1}{p}} (1-|\phi(z_n)|^2)^{\frac{N+1}{p}}} \ge n$$

for every $n \in \mathbf{N}$, which is a contradiction. ∎

The following lemma can be shown by standard arguments, see [9].

Lemma 4. *Let w and v be weights and the operator $\psi C_\phi : A_{v,p} \to H_w^\infty$ be bounded. Then ψC_ϕ is compact if and only if for any bounded sequence $(f_n)_n$ in $A_{v,p}$ which converges to zero uniformly on the compact subsets of B_N we have that $\|\psi C_\phi f_n\|_w \to 0$.*

Theorem 5. *Let w be a weight and v be a weight as in Lemma 1 such that there is $M > 0$ with $\sup_{\alpha \in B_N} \sup_{z \in B_N} \frac{v(z)}{|v_\alpha(z)|} \leq M$. Moreover, let $\phi : B_N \to B_N$ be analytic with $\|\phi\|_\infty = 1$ and $\psi \in H_w^\infty$. Furthermore, we assume that the operator $\psi C_\phi : A_{v,p} \to H_w^\infty$ is bounded. Then ψC_ϕ is compact if and only if*

$$\lim_{r \to 1} \sup_{\{z; |\phi(z)| > r\}} \frac{w(z)|\psi(z)|}{(1 - |\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} = 0. \qquad (*)$$

Proof. First, we assume that $(*)$ holds. Let $(f_n)_n$ be a bounded sequence in $A_{v,p}$ which converges to zero uniformly on the compact subsets of B_N. Let $K = \sup_n \|f_n\|_{v,p} < \infty$. Given $\varepsilon > 0$ there is $r > 0$ such that if $|\phi(z)| > r$, then

$$\frac{w(z)|\psi(z)|}{(1 - |\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} < \frac{\varepsilon \left(\int_{B_N} v(t)\, dV(t) \right)^{\frac{1}{p}}}{2KC^{\frac{1}{p}}}.$$

By Lemma 1 we have

$$|f_n(z)| \leq \frac{C^{\frac{1}{p}} \|f_n\|_{v,p}}{\left(\int_{B_N} v(t)\, dV(t) \right)^{\frac{1}{p}} (1 - |z|^2)^{\frac{N+1}{p}} v(z)^{\frac{1}{p}}}$$

for every $z \in B_N$ and every $n \in \mathbf{N}$. Thus, for $z \in B_N$ with $|\phi(z)| > r$, we obtain

$$\begin{aligned} w(z)|\psi C_\phi f_n(z)| &= w(z)|\psi(z)||f_n(\phi(z))| \\ &\leq \frac{C^{\frac{1}{p}} w(z)|\psi(z)|}{\left(\int_{B_N} v(t)\, dV(t) \right)^{\frac{1}{p}} (1 - |\phi(z)|^2)^{\frac{N+1}{p}} v(\phi(z))^{\frac{1}{p}}} \|f_n\|_{v,p} < \frac{\varepsilon}{2} \end{aligned}$$

for all n.

On the other hand, since $f_n \to 0$ uniformly on $\{u; |u| \leq r\}$ there is an $n_0 \in \mathbf{N}$ such that, if $|\phi(z)| \leq r$ and $n \geq n_0$, then $|f_n(\phi(z))| < \frac{\varepsilon}{2N}$, where $N = \sup_{z \in B_N} w(z)|\psi(z)| < \infty$ and hence

$$w(z)|\psi C_\phi f_n(z)| = w(z)|\psi(z)||f_n(\phi(z))| < \frac{\varepsilon}{2}$$

for every $z \in B_N$ with $|\phi(z)| \leq r$ and every $n \geq n_0$.

Conversely, suppose that $\psi C_\phi : A_{v,p} \to H_w^\infty$ is compact and that $(*)$ does not hold. Then there are $\delta > 0$ and $(z_n)_n \subset B_N$ with $|\phi(z_n)| \to 1$ such that

$$\frac{w(z_n)|\psi(z_n)|}{v(\phi(z_n))^{\frac{1}{p}} (1 - |\phi(z_n)|^2)^{\frac{N+1}{p}}} \geq \delta$$

for all n. For each n consider the function

$$f_n(z) := \frac{1}{v_{\phi(z_n)}(z)^{\frac{1}{p}}} \sigma_{\phi(z_n)}(z)^{\frac{N+1}{p}} \left(\frac{\langle z, \phi(z_n) \rangle}{|\phi(z_n)|} \right)^{\frac{1}{1-|\phi(z_n)|}} \text{ for every } z \in B_N.$$

Since $|\langle z, \phi(z_n)\rangle| \le |\phi(z_n)|$ for every $z \in B_N$, the sequence $(f_n)_n$ is norm bounded and $f_n \to 0$ pointwise. Thus, it follows that a subsequence of $(\psi C_\phi f_n)_n$ tends to 0 in H_w^∞. On the other hand

$$\begin{aligned} \|\psi C_\phi f_n\|_w &\ge w(z_n)|\psi C_\phi f_n(z_n)| = w(z_n)|\psi(z_n)||f_n(\phi(z_n))| \\ &= \frac{w(z_n)|\psi(z_n)|}{(1-|\phi(z_n)|^2)^{\frac{N+1}{p}} v(\phi(z_n))^{\frac{1}{p}}} \ge \delta, \end{aligned}$$

which is a contradiction. ∎

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Mathematical Institute, University of Paderborn, D-33095 Paderborn, Germany

E-mail: `lichte@math.uni-paderborn.de`