МАТЕМАТИЧКИ ВЕСНИК
63, 2 (2011), 79–86
June 2011

оригинални научни рад
research paper

ϕ-RECURRENT TRANS-SASAKIAN MANIFOLDS

H. G. Nagaraja

Abstract. In this paper we present some results for ϕ recurrent trans-Sasakian manifolds. We find conditions for such manifolds to be of constant curvature. Finally we give an example of a 3-dimensional ϕ- recurrent trans-Sasakian manifold.

1. Introduction

A class of almost contact metric manifolds known as trans-Sasakian manifolds was introduced by J. A. Oubina [6] in 1985. This class contains α-Sasakian, β-Kenmotsu and co-symplectic manifolds. An almost contact metric structure on a manifold M is called a trans-Sasakian structure if the product manifold $M \times R$ belongs to the class W_4, a class of Hermitian manifolds which are closely related to a locally conformal Kähler manifolds. Trans-Sasakian manifolds were studied extensively by J. C. Marrero [5], M. M. Tripathi [8], U. C. De [2, 3, 4] and others. M. M. Tripathi [8] proved that trans-Sasakian manifolds are always generalized quasi-Sasakian.

U. C. De et al. [2] generalized the notion of local ϕ-symmetry and introduced the notion of ϕ-recurrent Sasakian manifolds. In the present paper we study ϕ-recurrent trans-Sasakian manifolds. In Section 3, we prove that a conformally flat ϕ-recurrent trans-Sasakian manifold is a manifold of constant curvature. In the same section trans-Sasakian manifolds with η-parallel Ricci-tensor are considered and we prove that the scalar curvature of such a manifold is a constant. In Section 4, it is proved that a ϕ-recurrent conformally flat trans-Sasakian manifold is η-Einstein. Finally we construct an example of a 3-dimensional ϕ-recurrent trans-Sasakian manifold. This verifies the results proved in Section 3.

2. Preliminaries

Let M be a $(2n+1)$-dimensional almost contact metric manifold with an almost contact metric structure (ϕ, ξ, η, g), where ϕ, ξ, η are tensor fields on M of types

2010 AMS Subject Classification: 53C25.
Keywords and phrases: Trans-Sasakian; ϕ-recurrent; conformally flat; η-parallel; η-Einstein; constant curvature.

(1,1), (1,0), (0,1) respectively and g is the Riemannian metric on M such that

$$(a)\ \phi^2 = -I + \eta \otimes \xi,\ \ (b)\ \eta(\xi) = 1,\ \ (c)\ \ \phi(\xi) = 0,\ \ (d)\ \eta \circ \phi = 0 \tag{2.1}$$

The Riemanian metric g on M satisfies the condition

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y) \tag{2.2}$$

$$g(X, \phi Y) = -g(\phi X, Y) \tag{2.3}$$

$\forall X, Y \in TM$. An almost contact metric structure (ϕ, ξ, η, g) in M is called a trans-Sasakian structure [1] if the product manifold $(M \times R, J, G)$ belongs to the class W_4, where J is the complex structure on $(M \times R)$ defined by

$$J(X, \lambda\frac{d}{dt}) = (\phi - \lambda\xi, \eta(X)\frac{d}{dt}) \tag{2.4}$$

for all vector fields X on M and smooth functions λ on $(M \times R)$ and G is the product metric on $(M \times R)$. This may be expressed by the following condition [1]

$$(\nabla_X \phi)(Y) = \alpha(g(X, Y)\xi - \eta(Y)X) + \beta(g(\phi X, Y)\xi - \eta(Y)\phi X), \tag{2.5}$$

where α and β are smooth functions on M.

From (2.5), we have

$$(\nabla_X \xi) = -\alpha(\phi X) + \beta(X - \eta(X)\xi) \tag{2.6}$$

$$(\nabla_X \eta)(Y) = -\alpha g(\phi X, Y) + \beta(\phi X, \phi Y). \tag{2.7}$$

In a $(2n+1)$-dimensional trans-Sasakian manifold, from (2.5), (2.6), (2.7), we can derive [3]

$$\begin{aligned} R(X, Y)\xi = {} & (\alpha^2 - \beta^2)(\eta(Y)X - \eta(X)Y) + 2\alpha\beta(\eta(Y)\phi X - \eta(X)\phi Y) \\ & - (X\alpha)\phi Y + (Y\alpha)\phi X - (X\beta)\phi^2 Y + (Y\beta)\phi^2 X \end{aligned} \tag{2.8}$$

$$S(X, \xi) = (2n(\alpha^2 - \beta^2) - \xi\beta)\eta(X) - (2n-1)(X\beta) - (\phi X)\alpha. \tag{2.9}$$

Further we have

$$2\alpha\beta + \xi\alpha = 0. \tag{2.10}$$

In a conformally flat manifold the curvature tensor R satisfies

$$\begin{aligned} R(X, Y, Z, W) = {} & \frac{1}{2n-1}[S(Y, Z)g(X, W) + g(Y, Z)S(X, W) - S(X, Z)g(Y, W) \\ & - g(X, Z)S(Y, W)] - \frac{r}{2n(2n-1)}[g(Y, Z)g(X, W) - g(X, Z)g(Y, W)]. \end{aligned} \tag{2.11}$$

From (2.8) we have

$$R(\xi, X, Y, \xi) = (\alpha^2 - \beta^2 - \xi\beta)g(\phi X, \phi Y). \tag{2.12}$$

Suppose α and β are constants. Then from (2.9), (2.11), (2.12), we obtain

$$S(X, Y) = (\frac{r}{2n} - (\alpha^2 - \beta^2))g(\phi X, \phi Y) - 2n(\alpha^2 - \beta^2)\eta(X)\eta(Y). \tag{2.13}$$

Applying (2.13) in (2.11), we get

$$R(X,Y)Z = \frac{1}{2n-1}[(\frac{r}{2n} - 2(\alpha^2-\beta^2))(g(Y,Z)X - g(X,Z)Y) + (\frac{r}{2n} + (2n+1))(\alpha^2-\beta^2)\{(g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi) + (\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y)\}]. \quad (2.14)$$

From (2.10), for constants α and β, we have

Proposition 2.1. *A trans-Sasakian manifold with α and β are constants is either β-Sasakian or α-Kenmotsu or co-symplectic.*

It is well known that β-Sasakian manifolds are quasi Sasakian and α-Kenmotsu manifold are $C(-\alpha^2)$ manifolds. Hence we have the following corollary.

Corollary 2.1. *In a trans-Sasakian manifold M with α and β are constants, one of the following holds.*
(i) M is quasi Sasakian (ii) M is a $C(-\alpha^2)$ manifold (iii) M is co-symplectic.

3. Conformally flat ϕ-recurrent trans-Sasakian manifolds

Definition 3.1 A trans-Sasakian manifold is said to be ϕ-recurrent if

$$\phi^2(\nabla_W R)(X,Y)Z = A(W)R(X,Y)Z, \quad (3.1)$$

$\forall$ X,Y,Z,W $\in$ TM.

Differentiating (2.14) covariantly with respect to W, we get

$$(\nabla_W R)(X,Y)Z = \frac{1}{2n-1}[(\frac{dr(W)}{2n}(g(Y,Z)X - g(X,Z)Y) + (\frac{dr(W)}{2n}\{(g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi) + (\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y)\}] + [(\frac{r}{2n} + 3(\alpha^2-\beta^2) \{g(Y,Z)((\nabla_W\eta)(X)\xi - \eta(X)(\nabla_W\xi)) - g(X,Z)((\nabla_W\eta)(Y)\xi - \eta(Y)(\nabla_W\xi)) + (\nabla_W\eta)(Y)\eta(Z) + \eta(Y)(\nabla_W\eta)(Z) - (\nabla_W\eta)(X)\eta(Z) - \eta(X)(\nabla_W\eta)(Z)\}]. \quad (3.2)$$

We may assume that all vector fields X, Y, Z, W are orthogonal to ξ. Then (3.2) takes the form

$$(\nabla_W R)(X,Y)Z = \frac{1}{2n-1}[(\frac{dr(W)}{2n})(g(Y,Z)X - g(X,Z)Y) + (\frac{r}{2n} + 3(\alpha^2-\beta^2))\{g(Y,Z)(\nabla_W\eta)(X) - g(X,Z)(\nabla_W\eta)(Y)\}\xi]. \quad (3.3)$$

Applying ϕ^2 to both sides of (3.3), we get

$$A(W)R(X,Y)Z = \frac{1}{2n-1}[\frac{dr(W)}{2n}(g(Y,Z)X - g(X,Z)Y)]$$

i.e.

$$R(X,Y)Z = \frac{1}{2n(2n-1)}[\frac{dr(W)}{A(W)}(g(Y,Z)X - g(X,Z)Y)].$$

Putting $W = e_i$ in the above equation, where $\{e_i\}$ is an orthonormal basis of the tangent space at any point of the manifold and taking summation over i, $1 \leq i \leq 2n+1$, we obtain

$$R(X,Y)Z = \lambda(g(Y,Z)X - g(X,Z)Y),$$

where $\lambda = (\frac{dr(e_i)}{2n(2n-1)A(e_i)})$ is a scalar. Since A is non zero, λ will be a constant. Therefore M is of constant curvature λ. Thus we can state that

Theorem 3.1. *A conformally flat ϕ-recurrent trans-Sasakian manifold of dimension greater than 3 is a manifold of constant curvature provided α and β are constants.*

Since three dimensional Riemannian manifolds are conformally flat, we have

Corollary 3.1. *A three dimensional ϕ-recurrent trans-Sasakian manifold is a manifold of constant curvature.*

Now from Proposition 2.1 and the above corollary, we have

Corollary 3.2. *A three dimensional ϕ-recurrent β-Sasakian manifold (or α-Kenmotsu manifold or co-symplectic manifold) is a manifold of constant curvature.*

By virtue of (2.1)(a) and (3.1), we have

$$-(\nabla_W R)(X,Y)Z + \eta((\nabla_W R)(X,Y)Z)\xi = A(W)R(X,Y)Z$$

from which we get

$$-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) = A(W)R(X,Y,Z,U). \tag{3.4}$$

Putting $X = U = e_i$ and summing over $i = 1,\ldots,2n+1$, we get

$$-(\nabla_W S)(Y,Z) + \sum \eta((\nabla_W R)(e_i,Y)Z)\eta(e_i) = A(W)S(Y,Z). \tag{3.5}$$

The second term of (3.5) by putting $Z = \xi$ takes the form $g((\nabla_W R)(e_i,Y)\xi,\xi)g(e_i,\xi)$. Consider

$$\begin{aligned} g((\nabla_W R)(e_i,Y)\xi,\xi) = {} & g(\nabla_W R(e_i,Y)\xi,\xi) - g(R(\nabla_W e_i,Y)\xi,\xi) \\ & - g(R(e_i,\nabla_W Y)\xi,\xi) - g(R(e_i,Y)\nabla_W \xi,\xi) \end{aligned} \tag{3.6}$$

at $P \in M$.

Using (2.8), (2.1)(d) and $g(X,\xi) = \eta(X)$, we obtain

$$\begin{aligned} g(R(e_i,\nabla_W Y)\xi,\xi) = {} & g((\alpha^2-\beta^2)(\eta(\nabla_W Y)e_i - \eta(e_i)(\nabla_W Y)) \\ & + 2\alpha\beta(\eta(\nabla_W Y)\phi e_i - \eta(e_i)\phi(\nabla_W Y)) + (\nabla_W Y)\alpha)\phi e_i - \\ & (e_i\alpha)\phi(\nabla_W Y) - (e_i\beta)\phi^2(\nabla_W Y) + (\nabla_W Y\beta)\phi^2 e_i = 0. \end{aligned} \tag{3.7}$$

By virtue of $g(R(e_i,Y)\xi,\xi)=g(R(\xi,\xi)e_i,Y)=0$ and (3.7), (3.6) reduce to

$$g((\nabla_W R)(e_i,Y)\xi,\xi)=g(\nabla_W R(e_i,Y)\xi,\xi)-g(R(e_i,Y)\nabla_W\xi,\xi). \tag{3.8}$$

Since $(\nabla_X g)=0$, we have $g((\nabla_W R)(e_i,Y)\xi,\xi)+g(R(e_i,Y)\xi,\nabla_W\xi)=0$, which implies

$$g((\nabla_W R)(e_i,Y)\xi,\xi)=-g(R(e_i,Y)\xi,\nabla_W\xi)-g(R(e_i,Y)\nabla_W\xi,\xi). \tag{3.9}$$

Using (2.6) and by the skew symmetry of R, we get

$$\begin{aligned} &g((\nabla_W R)(e_i,Y)\xi,\xi)=\\ &=g(R(e_i,Y)\xi,-\alpha(\phi W)+\beta(W-\eta(W)\xi))+g(R(e_i,Y)-\alpha(\phi W)+\beta(W-\eta(W)\xi),\xi)\\ &=g(R(-\alpha(\phi W)+\beta(W-\eta(W)\xi,\xi)Y,e_i),))+g(R(\xi,-\alpha(\phi W)+\beta(W-\eta(W)\xi))Y,e_i).\end{aligned}$$

Multiplying the above equation by $\eta(e_i)=g(\xi,e_i)$ and summing over $i=1,\dots,$ $2n+1$, we get

$$\begin{aligned} &\sum\eta((\nabla_W R)(e_i,Y)Z)g(e_i,\xi)=\\ &\quad=\sum\{g(R(-\alpha(\phi W)+\beta(W-\eta(W)\xi,\xi)Y,e_i))g(e_i,\xi)+\\ &\quad g(R(\xi,-\alpha(\phi W)+\beta(W-\eta(W)\xi))Y,e_i)g(e_i,\xi)\}=\\ &\quad=\{g(R(-\alpha(\phi W)+\beta(W-\eta(W)\xi,\xi)Y,\xi))\\ &\qquad+g(R(\xi,-\alpha(\phi W)+\beta(W-\eta(W)\xi))Y,\xi)\}=0.\end{aligned}$$

Replacing Z by ξ in (3.5) and using (2.9) we get

$$-(\nabla_W S)(X,\xi)=A(W)\{2n(\alpha^2-\beta^2)\eta(X)\} \tag{3.10}$$

provided α and β are constants. Now from

$$(\nabla_X S)(Y,\xi)=\nabla_X S(Y,\xi)-S(\nabla_X Y,\xi)-S(Y,\nabla_X\xi).$$

Using (2.6) and (2.9), for constant α and β, we have

$$(\nabla_X S)(Y,\xi)=2n(\alpha^2-\beta^2)[(\nabla_X\eta)(Y)+\beta\eta(X)\eta(Y)]+S(Y,\alpha\phi X-\beta X). \tag{3.11}$$

From (3.11), (2.3) and (2.7), we obtain

$$(\nabla_X S)(Y,\xi)=2n(\alpha^2-\beta^2)[\beta g(X,Y)-\alpha g(X,\phi Y)]+S(Y,\alpha\phi X-\beta X). \tag{3.12}$$

From (3.10) and (3.12), we have

$$-A(X)\{2n(\alpha^2-\beta^2)\eta(Y)\}=2n(\alpha^2-\beta^2)[\beta g(X,Y)-\alpha g(X,\phi Y)]+S(Y,\alpha\phi X-\beta X). \tag{3.13}$$

Replacing Y by ϕY in (3.13) and using (2.2), we obtain

$$2n(\alpha^2-\beta^2)[\beta g(X,\phi Y)+\alpha g(X,Y)-\alpha\eta(X)\eta(Y)]+\alpha S(\phi Y,\phi X)-\beta S(\phi Y,X)=0$$

i.e.

$$-\alpha S(\phi Y,\phi X)+\beta S(\phi Y,X)=2n(\alpha^2-\beta^2)[\beta g(X,\phi Y)+\alpha g(\phi X,\phi Y)]. \tag{3.14}$$

Interchanging Y and X in (3.14) and by using the skew symmetry of ϕ, we obtain

$$-\alpha S(\phi X,\phi Y) = 2n\alpha(\alpha^2-\beta^2)(g(\phi X,\phi Y)). \tag{3.15}$$

By skew symmetry of ϕ and using (2.9), we obtain $S(\phi X,\phi Y) = -S(\phi^2 X, Y) = S(X,Y) - 2n(\alpha^2-\beta^2)\eta(X)\eta(Y)$. Substituting this in (3.15), we get

$$S(X,Y) = ag(X,Y) + b\eta(X)\eta(Y), \tag{3.16}$$

where $a = 2n(\alpha^2-\beta^2)$, i.e. M is η-Einstein. Thus we have

THEOREM 3.2 *A ϕ-recurrent conformally flat trans-Sasakian manifold is η-Einstein provided α and β are constants.*

COROLLARY 3.3 *A 3-dimensional ϕ-recurrent trans-Sasakian manifold is η-Einstein provided α and β are constants.*

4. Trans-Sasakian manifolds with η-parallel Ricci tensor

Let us consider a trans-Sasakian manifold M of dimension 2n+1 with η- parallel Ricci tensor. Replacing Y by ϕY and Z by ϕZ in (2.13), we obtain

$$S(\phi X,\phi Y) = (\frac{r}{2n} - (\alpha^2-\beta^2))(g(X,Y) - \eta(X)\eta(Y). \tag{4.1}$$

Differentiating (4.1) covariantly with respect to X we obtain

$$\begin{aligned}(\nabla_X S)(\phi Y,\phi Z) = {}& (\frac{dr(X)}{2n})(g(Y,Z)X - \eta(Y)\eta(Z))\\ & -(\frac{r}{2n} - (\alpha^2-\beta^2))\{(\nabla_X\eta)(Y)\eta(Z) + \eta(Y)(\nabla_X\eta)(Z)\}].\end{aligned} \tag{4.2}$$

Suppose the Ricci tensor is η-parallel. Then we obtain

$$(\frac{dr(X)}{2n})(g(Y,Z)-\eta(Y)\eta(Z)) = [(\frac{r}{2n}-(\alpha^2-\beta^2))\{(\nabla_X\eta)(Y)\eta(Z)+\eta(Y)(\nabla_X\eta)(Z)\}]. \tag{4.3}$$

Putting $Y = Z = e_i$ in (4.3), where $\{e_i\}$ is an orthonormal basis and summing over i=1, … ,2n+1, we obtain

$$dr(X) = (\frac{r}{2n} - 2(\alpha^2-\beta^2))(\nabla_X\eta)(\xi). \tag{4.4}$$

Since $\eta(\xi) = 1$, from (2.5), we have $(\nabla_X\eta)(\xi) = 0$. Thus from (4.4), we obtain $dr(X) = 0$ or r is a constant. Thus we have

THEOREM 4.1. *In a conformally flat trans-Sasakian manifold with η-parallel Ricci tensor, the scalar curvature is constant provided α and β are constants.*

COROLLARY 4.1. *A 3-dimensional trans-Sasakian manifold with η-parallel Ricci tensor, the scalar curvature is constant provided α and β are constants.*

5. Example of ϕ-recurrent trans-Sasakian manifolds

Consider three dimensional manifold $M = \{(x, y, z) \in R^3 \setminus z \neq 0\}$, where (x, y, z) are the standard coordinates of R^3. The vector fields

$$e_1 = \frac{x}{z}\frac{\partial}{\partial x}, \quad e_2 = \frac{y}{z}\frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z} \tag{5.1}$$

are linearly independent at each point of M. Let g be the Riemannian metric defined by

$$g(e_1, e_1) = 1, g(e_2, e_2) = 1, g(e_3, e_3) = 1, g(e_1, e_2) = 0, g(e_1, e_3) = 0, g(e_2, e_3) = 0. \tag{5.2}$$

Let η be the 1-form defined by $\eta(X) = g(X, \xi)$ for any vector field X. Let ϕ be the $(1, 1)$ tensor field defined by

$$\phi(e_1) = e_2, \phi(e_2) = -e_1, \phi(e_3) = 0. \tag{5.3}$$

Then by using the linearity of ϕ and g we have $\phi^2 X = -X + \eta(X)\xi$, with $\xi = e_3$.

Further $g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$ for any vector fields X and Y. Hence for $e_3 = \xi$, the structure defines an almost contact structure on M. Let ∇ be the Levi-Civita connection with respect to the metric g, then we have

$$[e_1, e_2] = 0, [e_1, e_3] = \frac{1}{z}e_1, [e_2, e_3] = \frac{1}{z}e_2. \tag{5.4}$$

The Riemannian connection ∇ of the metric g is given by

$$\begin{aligned} 2g(\nabla_X Y, Z) &= Xg(Y, Z) + Yg(Z, X) - Zg(X, Y) \\ &\quad - g(X, [Y, Z]) - g(Y, [X, Z]) + g(Z, [X, Y]). \end{aligned} \tag{5.5}$$

Using (5.5), we have

$$2g(\nabla_{e_1} e_3, e_1) = 2g(\frac{1}{z}e_1, e_1) + 2g(e_2, e_1) = 2g(\frac{1}{z}e_1 + e_2, e_1),$$

since $g(e_1, e_2) = 0$. Thus

$$\nabla_{e_1} e_3 = \frac{1}{z}e_1 + e_2. \tag{5.6}$$

Again by (5.5) we get,

$$2g(\nabla_{e_2} e_3, e_2) = 2g(\frac{1}{z}e_2, e_2) - 2g(e_1, e_2) = 2g(\frac{1}{z}e_2 - e_1, e_2),$$

since $g(e_1, e_2) = 0$. Therefore we have

$$\nabla_{e_2} e_3 = \frac{1}{z}e_2 - e_1. \tag{5.7}$$

Again from (5.5) we have

$$\begin{gathered} \nabla_{e_3} e_3 = 0, \quad \nabla_{e_1} e_1 = \frac{-1}{z}e_1, \quad \nabla_{e_1} e_2 = 0, \\ \nabla_{e_2} e_1 = 0, \quad \nabla_{e_2} e_1 = 0, \quad \nabla_{e_2} e_2 = -\frac{1}{z}e_2, \\ \nabla_{e_3} e_1 = 0, \quad \nabla_{e_3} e_2 = 0. \end{gathered} \tag{5.8}$$

The manifold M satisfies (2.5) with $\alpha = -1$ and$\beta = \frac{1}{z}$. Hence M is a trans-Sasakian manifold. Using the relations (5.6), (5.7) and (5.8), the non-vanishing components of the curvature tensor are computed as follows:

$$\begin{aligned} R(e_1, e_3)e_3 &= \frac{1}{z^2}e_1, \quad R(e_3, e_1)e_3 = -\frac{1}{z^2}e_1, \\ R(e_2, e_3)e_3 &= \frac{1}{z^2}e_2, \quad R(e_3, e_2)e_3 = -\frac{1}{z^2}e_2. \end{aligned} \tag{5.9}$$

The vectors $\{e_1, e_2, e_3\}$ form a basis of M and so any vector X can be written as $X = a_1e_1 + a_2e_2 + a_3e_3$ where $a_i \in R^+, i = 1, 2, 3$. From (5.9), we have

$$(\nabla_X R)(e_1, e_3)e_3 = -\frac{2a_3}{z^3}e_1$$

and

$$(\nabla_X R)(e_2, e_3)e_3 = -\frac{2a_3}{z^3}e_2.$$

Applying ϕ^2 to both sides of the above equations and using (5.3), we obtain

$$\phi^2((\nabla_X R)(e_1, e_3)e_3) = A(X)R(e_1, e_3)e_3$$

and

$$\phi^2((\nabla_X R)(e_2, e_3)e_3) = A(X)R(e_2, e_3)e_3,$$

where $A(X) = \frac{2a_3}{z}$ is a non vanishing 1-form. This implies that there exists a ϕ-recurrent trans-Sasakian manifold of dimension 3.

From the non vanishing curvature components as given in (5.9), we have

$$R(e_1, e_3)e_3 = \lambda(g(e_3, e_3)e_1 - g(e_1, e_3)e_3)$$

and

$$R(e_2, e_3)e_3 = \lambda(g(e_3, e_3)e_2 - g(e_2, e_3)e_3).$$

This verifies Corollary 3.2.

REFERENCES

[**1**] D.E. Blair, J.A. Oubina, *Conformal and related changes of metric on the product of two almost contact metric manifolds*, Publ. Matematiques **34** (1990), 199–207.

[**2**] U.C. De, A.A. Shaikh, S. Biswas, *On ϕ-recurrent Sasakian manifolds*, Novi Sad J. Math. **33** (2003), 43–48.

[**3**] U.C. De, M.M. Tripathi, *Ricci tensor in 3-dimensional trans-Sasakian manifolds*, Kyungpook Math. J. **43** (2003), 247–255.

[**4**] U.C. De, A. Yildiz, A.F. Yaliniz, *Locally ϕ-symmetric normal almost contact metric manifolds of dimension* 3, Appl. Math. Lett. **22** (2009), 723–727.

[**5**] J.C. Marrero, *The local structure of trans-Sasakian manifolds*, Ann. Mat. Pura. Appl. **162** (1992), 77–86.

[**6**] J.A. Oubina, *New classes of almost contact metric structures*, Publ. Math. Debrecen, **32** (1985), 187–193.

[**7**] T. Takahashi, *Sasakian ϕ-symmetric spaces*, Tohoku Math. J. **29** (1977), 91–113.

[**8**] M.M. Tripathi, *Trans-Sasakian manifolds are generalised quasi-Sasakian*, Nepali Math. Sci. Rep. **18** (1999-2000), No.1-2, (2001), 11–14.

(received 11.11.2009; in revised form 29.11.2010)

Department of Mathematics, Central College, Bangalore University, Bangalore-560 001, India
E-mail: hgnraj@yahoo.com