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EXISTENCE OF MILD SOLUTIONS OF A SEMILINEAR NONCONVEX DIFFERENTIAL INCLUSION WITH NONLOCAL CONDITIONS

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Abstract. We show two existence results of a mild solution for a semilinear nonconvex differential inclusion, with nonlocal condition, governed by a family of linear operators, not necessarily bounded or closed.

1. Introduction

The aim of this paper is to establish two existence results of mild solutions of the following semilinear differential inclusion:

$$\begin{cases} \dot{x}(t) \in A(t)x(t) + F(t, x(t)) \colon \text{ a.e. on } [0, T]; \\ x(0) = g(x(\cdot)). \end{cases} \tag{1.1}$$

where $F\colon [0,T] \times E \to 2^E$ is a nonconvex or noncompact multi-valued map, $\{A(t) : t \in [0,T]\}$ is a family of densely defined linear operators not necessarily bounded or closed, $g : \mathcal{C}([0,T], E) \to E$ is a function and E is a Banach space.

For review of results on semilinear differential equations with nonlocal conditions, we refer the reader to the papers by Byszewski [3, 4, 5], by Liany, Liu and Xiao [12], by Xue [15], by Fan, Dong and Li [9], and the references cited therein. Existence results for semilinear differential inclusions received much attention in the recent years. Cardinali and Rubbioni [6] have studied semilinear differential inclusions with initial conditions, where the set-valued map is a compact and convex values. This last cited work contains the analogous results provided by Kamenskii, Obukhowskii and Zecca [11] for inclusions with constant operator. Al-Omair and G. Ibrahim [1] employ the methods of Kamenskii, Obukhowskii and Zecca, as well as Cardinali and Rubbioni to prove the existence of mild solution for (1.1) without compactness assumption on the evolution operator $T(\cdot, \cdot)$ which is generated by the

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family $\{A(t) : t \in [0,T]\}$. The authors assumed that the set-valued map F is a closed and convex values and satisfies a compactness condition involving the Hausdorff measure of noncompactness. The function g is continuous and completely continuous.

In this paper, we prove two existence results of mild solution for (1.1) governed by a family of linear operators, not necessarily bounded or closed. The set-valued map F is not convex and not compact in the first case and not convex in the second case. The function g is not completely continuous, it is Lipschitz continuous in the first case and continuous in the second case. No compactness condition involving the Hausdorff measure of noncompactness is assumed on F.

2. Preliminaries and notations

Let E be a real Banach space with the norm $\|\cdot\|$, $I = [0,T]$ and $T > 0$. We denote by $\mathcal{C}([0,T],E)$ the Banach space of continuous functions from $[0,T]$ to E with the norm $\|x(\cdot)\|_\infty := \sup\big\{\|x(t)\|; t \in [0,T]\big\}$ and by $\mathcal{L}(E)$ the space of bounded linear operators on E. Let $\{A(t) : t \in I\}$ be a family of densely defined linear operators (not necessarily bounded or closed) on E and $T\colon \Delta = \big\{(t,s) : 0 \le s \le t \le T\big\} \to \mathcal{L}(E)$ be the evolution operator generated by the family $\{A(t) : t \in I\}$. We say that a subset A of $[0,T] \times E$ is $\mathcal{L}\otimes\mathcal{B}$-measurable if A belongs to the σ-algebra generated by all sets of the form $I \times D$, where I is Lebesgue measurable in $[0,T]$ and D is measurable in E. For $x \in E$ and for nonempty subsets A, B of E, we denote $d(x,A) = \inf\{d(x,y); y \in A\}$, $e(A,B) := \sup\{d(x,B); x \in A\}$ and $H(A,B) := \max\{e(A,B), e(B,A)\}$. A multifunction is said to be measurable if its graph is measurable. For more details on measurable multifunction, we refer the reader to the book of Castaing-Valadier [7].

Now, let for every $t \in I$, $A(t)\colon E \to E$ be a linear operator such that

(i) For all $t \in I$, $D(A(t)) = D(A)$ and $\overline{D(A)} = E$.

(ii) For each $s \in I$ and each $x \in E$ there is a unique solution $v\colon [s,T] \to E$ for the evolution equation

$$\begin{aligned} v'(t) &= A(t)v(t), \quad t \in [s,T] \\ v(s) &= x. \end{aligned} \tag{2.1}$$

In this case an operator $T(\cdot,\cdot)$ can be defined as

$$T\colon \Delta = \{(t,s) : 0 \le s \le t \le T\} \to \mathcal{L}(E), \quad T(t,s)(x) = v(t),$$

where v is the unique solution of (2.1). The operator $T(\cdot,\cdot)$ is called the evolution operator generated by the family $\{A(t) : t \in I\}$. It is known that (see [13]) each operator $T(t,s)$ is strongly differentiable and such that $T(s,s) = I_E$, $T(t,r)T(r,s) = T(t,s)$ for all $0 \le s \le r \le t \le b$,

$$\frac{\partial T(t,s)}{\partial t} = A(t)T(t,s) \quad \text{and} \quad \frac{\partial T(t,s)}{\partial s} = -T(t,s)A(s).$$

Along this work, we assume that there exists $M > 0$ such that

$$\|T(t,s)\|_{\mathcal{L}(E)} \le M, \quad \forall (t,s) \in \Delta.$$

Definition 2.1. By a mild solution of problem (1.1), we mean a continuous function $x(\cdot)\colon I \to E$ such that

$$x(t) = T(t,0)g(x) + \int_0^t T(t,s)f(s)\,ds, \quad t \in I$$

where f is an integrable function such that $f(t) \in F(t, x(t))$, for almost every $t \in I$.

3. The Lipschitz case

In this section, our main purpose is to obtain the existence of a mild solution to (1.1), in the case when $F(\cdot,\cdot)$ is a closed multifunction, measurable in t and Lipschitz continuous in x. We use the fixed point theorem introduced by Covitz and Nadler for contraction multi-valued maps.

Definitions 3.1. Let $G\colon E \to 2^E$ be a multifunction with closed values.

(1) G is k-Lipschitz if

$$H\big(G(x), G(y)\big) \le k d(x,y), \quad \text{for each} \quad x, y \in E.$$

(2) G is a contraction if it is k-Lipschitz with $k < 1$.

(3) G has a fixed point if there exists $x \in E$ such that $x \in G(x)$.

Let us recall the following results that will be used in the sequel.

Lemma 3.2. [8] *If $G\colon E \to 2^E$ is a contraction with nonempty closed values, then it has a fixed point.*

Lemma 3.3. [16] *Assume that $F\colon [a,b] \times E \to 2^E$ is a multifunction with nonempty closed values satisfying:*

- *For every $x \in E$, $F(\cdot, x)$ is measurable on $[a,b]$;*
- *For every $t \in [a,b]$, $F(t,\cdot)$ is (Hausdorff) continuous on E.*

Then, for any measurable function $x(\cdot)\colon [a,b] \to E$, the multifunction $F(\cdot, x(\cdot))$ is measurable on $[a,b]$.

Definition 3.4. A measurable multi-valued function $F\colon [a,b] \to 2^E$ is said to be integrably bounded if there exists a function $h \in L^1([a,b], E)$ such that for all $v \in F(t)$, $\|v\| \le h(t)$ for almost every $t \in [a,b]$.

We shall prove the following theorem.

Theorem 3.5. *Let $g\colon \mathcal{C}([0,T], E) \to E$ be a λ-Lipschitz function and $F\colon [0,T] \times E \to 2^E$ be a set-valued map with nonempty closed values satisfying*

(i) *For each $x \in E$, $t \mapsto F(t,x)$ is measurable and integrably bounded;*

(ii) *There exists a function $m(\cdot) \in L^1([0,T], \mathbb{R}^+)$ such that for all $t \in [0,T]$ and for all $x_1, x_2 \in E$,*

$$H\big(F(t,x_1), F(t,x_2)\big) \le m(t)\|x_1 - x_2\|.$$

Then, if $M(\lambda + L(T)) < 1$, the problem (1.1) has at least one mild solution on $[0,T]$, where $L(T) = \int_0^T m(s)\,ds$.

Proof. For $y(\cdot) \in \mathcal{C}([0,T],E)$, set

$$S_{F,y(\cdot)} := \Big\{ f \in L^1([0,T],E) : f(t) \in F(t,y(t)) \text{ for a.e. } t \in [0,T] \Big\}.$$

By Lemma 3.3, for $y(\cdot) \in \mathcal{C}([0,T],E)$, $F(\cdot,y(\cdot))$ is closed and measurable, then it has a measurable selection which, by hypothesis (i), belongs to $L^1([0,T],E)$. Thus $S_{F,y(\cdot)}$ is nonempty. Let us transform the problem into a fixed point problem. Consider the multivalued map, $G\colon \mathcal{C}([0,T],E) \to 2^{\mathcal{C}([0,T],E)}$ defined as follows, for $y(\cdot) \in L^1([0,T],E)$, $G(y(\cdot))$ is the set of all $z(\cdot) \in \mathcal{C}([0,T],E)$, such that

$$z(t) = T(t,0)g(y(\cdot)) + \int_0^t T(t,s)f(s)\,ds,$$

where $f \in S_{F,y(\cdot)}$. We shall show that G satisfies the assumptions of Lemma 3.2. The proof will be given in two steps:

Step 1. *G has non-empty closed-values.* Indeed, let $(y_p(\cdot))_{p\geq 0} \in G(y(\cdot))$ converges to $\bar{y}(\cdot)$ in $\mathcal{C}([0,T],E)$. Then $\bar{y}(\cdot) \in \mathcal{C}([0,T],E)$ and for each $t \in [0,T]$,

$$y_p(t) \in T(t,0)g(y(\cdot)) + \int_0^t T(t,s)F(s,y(s))\,ds.$$

where

$$\int_0^t T(t,s)F(s,y(s))\,ds$$

is the Aumann integral of $T(t,\cdot)F(\cdot,y(\cdot))$, which is defined as

$$\int_0^t T(t,s)F(s,y(s))\,ds = \left\{ \int_0^t T(t,s)f(s)\,ds,\ f \in S_{F,y(\cdot)} \right\}.$$

Since the set

$$\int_0^t T(t,s)F(s,y(s))\,ds$$

is closed for all $t \in [0,T]$, we have

$$\bar{y}(t) \in T(t,0)g(y(\cdot)) + \int_0^t T(t,s)F(s,y(s))\,ds.$$

Then $\bar{y}(\cdot) \in G(y(\cdot))$. So $G(y(\cdot))$ is closed for each $y(\cdot) \in \mathcal{C}([0,T],E)$.

Step 2. *G is a contraction.* Indeed, let $y_1(\cdot), y_2(\cdot) \in \mathcal{C}([0,T],E)$ and $z_1(\cdot) \in G(y_1(\cdot))$. Then

$$z_1(t) = T(t,0)g(y_1(\cdot)) + \int_0^t T(t,s)f_1(s)\,ds,$$

where $f_1 \in S_{F,y_1(\cdot)}$. Let $\varepsilon > 0$. Consider the multivalued map $U_\varepsilon\colon [0,T] \to 2^E$, defined by

$$U_\varepsilon(t) = \big\{ x \in E : \|f_1(t) - x\| \leq m(t)\|y_1(t) - y_2(t)\| + \varepsilon \big\}.$$

For each $t \in [0,T]$, $U_\varepsilon(t)$ is nonempty. Indeed, let $t \in [0,T]$. We have

$$H\big(F(t,y_1(t)),F(t,y_2(t))\big) \le m(t)\|y_1(t)-y_2(t)\|.$$

Hence, there exists $x \in F(t,y_2(t))$, such that

$$\|f_1(t)-x\| \le m(t)\|y_1(t)-y_2(t)\|+\varepsilon.$$

By Proposition III.4 in [7], the multifunction

$$V: t \to U_\varepsilon(t) \cap F(t,y_2(t)) \tag{3.1}$$

is measurable. Then there exists a measurable selection for V denoted f_2 such that, for all $t \in [0,T]$, $f_2(t) \in F(t,y_2(t))$ and

$$\|f_1(t)-f_2(t)\| \le m(t)\|y_1(t)-y_2(t)\|+\varepsilon.$$

Now, set for all $t \in [0,T]$,

$$z_2(t) = T(t,0)g(y_2(\cdot)) + \int_0^t T(t,s)f_2(s)\,ds.$$

Then

$$\begin{aligned}\|z_1(t)-z_2(t)\| &\le \|T(t,0)\|_{\mathcal{L}(E)}\|g(y_1(\cdot))-g(y_2(\cdot))\| \\ &\quad + \int_0^t \|T(t,s)\|_{\mathcal{L}(E)}\|f_1(s)-f_2(s)\|\,ds \\ &\le M\lambda\|y_1(\cdot)-y_2(\cdot)\|_\infty + M\|y_1(\cdot)-y_2(\cdot)\|_\infty\int_0^t m(s)\,ds + MT\varepsilon \\ &\le M(\lambda+L(T))\|y_1(\cdot)-y_2(\cdot)\|_\infty + MT\varepsilon.\end{aligned}$$

So, we conclude that

$$\|z_1(\cdot)-z_2(\cdot)\|_\infty \le M(\lambda+L(T))\|y_1(\cdot)-y_2(\cdot)\|_\infty + MT\varepsilon.$$

By the analogous relation, obtained by interchanging the roles of $y_1(\cdot)$ and $y_2(\cdot)$, it follows that

$$H\big(G(y_1(\cdot)),G(y_2(\cdot))\big) \le M(\lambda+L(T))\|y_1(\cdot)-y_2(\cdot)\|_\infty + MT\varepsilon.$$

By letting $\varepsilon \to 0$, we get

$$H\big(G(y_1(\cdot)),G(y_2(\cdot))\big) \le M(\lambda+L(T))\|y_1(\cdot)-y_2(\cdot)\|_\infty.$$

Consequently, G is a contraction. Hence, by Lemma 3.2, G has a fixed point $y(\cdot)$ which is a solution of (1.1). ∎

4. The lower semicontinuous case

In the sequel, we prove the existence of solutions of the problem (1.1), in the case where the set-valued maps is lower semicontinuous. We use Schaefer's fixed point theorem combined with a selection theorem of Bressan and Colombo (see [2]), for lower semicontinuous and nonconvex multi-valued operators with decomposable values. In this section, we assume that $T(t,s)$ is compact for $t-s>0$.

Definition 4.1. A subset B of $L^1([0,T],E)$ is decomposable if for all $u(\cdot),v(\cdot)\in B$ and $I\subset[0,T]$ measurable, the function $u(\cdot)\chi_I(\cdot)+v(\cdot)\chi_{[0,T]\setminus I}(\cdot)\in B$, where $\chi(\cdot)$ denotes the characteristic function.

Definitions 4.2. Let X be a nonempty closed subset of E and $G\colon X\to 2^E$ be a multi-valued operator with nonempty closed values. We say that:

- G is lower semi-continuous if the set $\{x\in X : G(x)\cap C\neq\emptyset\}$ is open for any open set C in E.
- G is completely continuous if $G(B)$ is relatively compact for every B bounded set of X.

Definition 4.3. Let $F\colon[0,T]\times E\to 2^E$ be a multi-valued map with nonempty compact values. Assign to F the multi-valued operator

$$\mathcal{F}\colon \mathcal{C}([0,T],E)\to 2^{L^1([0,T],E)},$$

defined by

$$\mathcal{F}(x(\cdot))=\Big\{y(\cdot)\in L^1([0,T],E) : y(t)\in F(t,x(t)) \text{ for a.e. } t\in[0,T]\Big\}.$$

The operator $\mathcal{F}$ is called the Niemytzki operator associated with F. We say F is the lower semi-continuous type if its associated Niemytzki operator $\mathcal{F}$ is lower semi-continuous and has nonempty closed and decomposable values.

Let us recall the following result that will be used in the sequel.

Lemma 4.4. [2] *Let E be a separable metric space and let $G\colon E\to 2^{L^1([0,T],E)}$ be a multi-valued operator which is lower semi-continuous and has nonempty closed and decomposable values. Then G has a continuous selection, i.e. there exists a continuous function $f\colon E\to L^1([0,T],E)$ such that $f(y)\in G(y)$ for every $y\in E$.*

We shall prove the following result.

Theorem 4.5. *Let $g\colon\mathcal{C}([0,T],E)\to E$ be a continuous function and $F\colon[0,T]\times E\to 2^E$ be a set-valued map with nonempty compact values satisfying*

(i) *$(t,x)\mapsto F(t,x)$ is $\mathcal{L}\otimes\mathcal{B}$-measurable;*

(ii) *$x\mapsto F(t,x)$ is lower semi-continuous for almost all $t\in[0,T]$;*

(iii) *there exists a function* $m(\cdot) \in L^1([0,T],\mathbb{R}^+)$ *such that for almost all* $t \in [0,T]$ *and all* $x \in E$

$$\|F(t,x)\| := \sup\Big\{\|y\| : y \in F(t,x)\Big\} \leq m(t).$$

(iv) *There exist positive constants* c *and* d *such that*

$$\|g(x)\| \leq c\|x(\cdot)\|_\infty + d, \quad \forall x(\cdot) \in \mathcal{C}([0,T],E).$$

(v) *For each bounded* $D \subset \mathcal{C}([0,T],E)$ *and* $t \in [0,T]$ *the set*

$$\Big\{T(t,0)g(y(\cdot)) + \int_0^t T(t,s)f(y(\cdot))(s)\,ds,\ y(\cdot) \in D\Big\}$$

is relatively compact, where $f\colon \mathcal{C}([0,T],E) \to L^1([0,T],E)$ *such that* $f(y(\cdot)) \in \mathcal{F}(y(\cdot))$ *for all* $y(\cdot) \in \mathcal{C}([0,T],E)$.

Then, if $1 - Mc > 0$, *the problem (1.1) has at least one mild solution on* $[0,T]$.

Proof. Remark that, by hypotheses, F is of lower semicontinuous type (see [10]). Then, by Lemma 4.4, there exists a continuous function $f\colon \mathcal{C}([0,T],E) \to L^1([0,T],E)$ such that $f(y(\cdot)) \in \mathcal{F}(y(\cdot))$ for all $y(\cdot) \in \mathcal{C}([0,T],E)$. Consider the problem:

$$\begin{cases} \dot{y}(t) = A(t)y(t) + f(y(\cdot))(t) \ \text{ a.e.;} \\ y(0) = g(y(\cdot)). \end{cases} \tag{4.1}$$

Remark that, if $y(\cdot) \in \mathcal{C}([0,T],E)$ is a solution of the problem (4.1) , then $y(\cdot)$ is a solution of the problem (1.1). Let us transform the problem (4.1) into a fixed point problem. Consider the operator, $G\colon \mathcal{C}([0,T],E) \to \mathcal{C}([0,T],E)$ defined as follows, for all $y(\cdot) \in \mathcal{C}([0,T],E)$ and for all $t \in [0,T]$:

$$G(y(\cdot))(t) = T(t,0)g(y(\cdot)) + \int_0^t T(t,s)f(y(\cdot))(s)\,ds.$$

We shall show that G has a fixed point. The proof will be given in several steps:

Step 1. *G is continuous.* Indeed, let $(y_p(\cdot))_{p\geq 0}$ converges to $y(\cdot)$ in $\mathcal{C}([0,T],E)$. Then for each $t \in [0,T]$

$$\begin{aligned}
&\big\|G(y_p(\cdot))(t) - G(y(\cdot))(t)\big\| \\
&\leq \|T(t,0)\|_{\mathcal{L}(E)}\|g(y_p(\cdot)) - g(y(\cdot))\| + \int_0^t \|T(t,s)\|_{\mathcal{L}(E)}\|f(y_p(\cdot))(s) - f(y(\cdot))(s)\|\,ds \\
&\leq M\|g(y_p(\cdot)) - g(y(\cdot))\| + M\int_0^T \|f(y_p(\cdot))(s) - f(y(\cdot))(s)\|\,ds.
\end{aligned}$$

By the continuity of g and f, it is easy to deduce that G is continuous.

Step 2. *G is bounded on bounded sets of* $\mathcal{C}([0,T],E)$. Indeed, it is sufficient to show that $G(B_r)$ is bounded for all $r \geq 0$, where $B_r = \{y(\cdot) \in \mathcal{C}([0,T],E) :$

$\|y(\cdot)\|_\infty \leq r\}$. Let $h \in G(B_r)$. For all $t \in [0,T]$ we have

$$\begin{aligned}\|h(t)\| &\leq \|T(t,0)\|_{\mathcal{L}(E)} \|g(y(\cdot))\| + \int_0^t \|T(t,s)\|_{\mathcal{L}(E)} \|f(y(\cdot))(s)\|\, ds \\ &\leq M(c\|y(\cdot)\|_\infty + d) + M \int_0^t m(s)\, ds \\ &\leq M(cr+d) + M\int_0^T m(s)\, ds.\end{aligned}$$

Then

$$\|h\|_\infty \leq M(cr+d) + M\int_0^T m(s)\, ds.$$

Hence $G(B_r) \subset B_\delta$, where δ is the right-hand side in the above inequality.

Step 3. *G sends bounded sets of $\mathcal{C}([0,T],E)$ into equicontinuous sets.* Indeed, let $h \in G(B_r)$. Then $h = G(y(\cdot))$ where $y(\cdot) \in B_r$. Let $t, s \in [0,T]$ such that $t < s$. We have

$$\begin{aligned}&\|h(s) - h(t)\| \\ &\quad\leq \|T(s,0) - T(t,0)\|_{\mathcal{L}(E)} \|g(y(\cdot))\| + \int_t^s \|T(s,\tau)\|_{\mathcal{L}(E)} \|f(y(\cdot))(\tau)\|\, d\tau \\ &\quad\quad + \int_0^t \|T(t,\tau) - T(s,\tau)\|_{\mathcal{L}(E)} \|f(y(\cdot))(\tau)\|\, d\tau \\ &\quad\leq (cr+d)\|T(s,0) - T(t,0)\|_{\mathcal{L}(E)} + M\int_t^s m(\tau)\, d\tau \\ &\quad\quad + \int_0^T \|T(t,\tau) - T(s,\tau)\|_{\mathcal{L}(E)} m(\tau)\, d\tau.\end{aligned}$$

The right-hand side of the above inequality tends to 0 as s converges to t, since $T(t,s)$ is a strongly continuous operator and the compactness of $T(t,s)$ for $t > s$ implies the continuity in the uniform operator topology (see [13]).

Step 4. *The following set is bounded*

$$\Omega = \Big\{ y(\cdot) \in \mathcal{C}([0,T],E) : \lambda y(\cdot) = G(y(\cdot)),\ \text{for some } \lambda > 1 \Big\}.$$

Indeed, let $y(\cdot) \in \Omega$. Then

$$y(t) = \lambda^{-1} T(t,0) g(y(\cdot)) + \lambda^{-1} \int_0^t T(t,s) f(y(\cdot))(s)\, ds.$$

So, we conclude that

$$\|y(\cdot)\|_\infty \leq \lambda^{-1} M(c\|y(\cdot)\|_\infty + d) + \lambda^{-1} M \int_0^T m(s)\, ds.$$

So, we get

$$(1 - \lambda^{-1} Mc)\|y(\cdot)\|_\infty \leq \lambda^{-1} Md + \lambda^{-1} M \int_0^T m(s)\, ds.$$

Since $1-\lambda^{-1}Mc > 1-Mc$, we obtain

$$(1-Mc)\|y(\cdot)\|_\infty \le \lambda^{-1}Md + \lambda^{-1}M\int_0^T m(s)\,ds.$$

Hence

$$\|y(\cdot)\|_\infty \le \frac{\lambda^{-1}Md}{1-Mc} + \frac{\lambda^{-1}M}{1-Mc}\int_0^T m(s)\,ds.$$

This shows that Ω is bounded.

In conclusion, by the Steps 1, 2, 3 and the hypothesis (v) combined with the Arzela-Ascoli theorem, we can conclude that G is completely continuous. Then by Schaefer's theorem (see [14], p. 29), we deduce that G has a fixed point which is a solution of (4.1). ■

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