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FIXED POINT THEOREMS FOR OCCASIONALLY WEAKLY COMPATIBLE MAPPINGS IN MENGER SPACES

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Abstract. In this paper, we prove common fixed point theorems for occasionally weakly compatible maps in Menger spaces. Our results never require the completeness of the whole space, continuity of the involved maps and containment of ranges amongst involved maps.

1. Introduction

The notion of probabilistic metric space (shortly PM-space), as a generalization of metric spaces, with non-deterministic distance, was introduced by K. Menger [15] in 1942. Since then the theory of PM-spaces has developed in many directions [19, 20]. A probabilistic generalization of metric spaces appears to be of interest in the investigation of physical quantities and physiological thresholds. It is also of a fundamental importance in probabilistic functional analysis. In 1972, V. M. Sehgal and A. T. Bharucha-Reid [21] initiated the study of contraction mappings in PM-spaces which is an important step in the development of fixed point theorems.

Various mathematicians weakened the notion of commutativity by introducing the notions of weak commutativity [22], compatibility [10] and weak compatibility [11] in metric spaces and proved a number of fixed point theorems using these notions. Recently, Al-Thagafi and Shahzad [3] weakened the notion of weakly compatible maps by introducing occasionally weakly compatible maps. It is worth to mention that every pair of commuting self-maps is weakly commuting, each pair of weakly commuting self-maps is compatible, each pair of compatible self-maps is weak compatible and each pair of weak compatible self-maps is occasionally weak compatible but the reverse is not always true. Many authors formulated the definitions of weakly commuting [24], compatible [16], weakly compatible maps [23] and occasionally weakly compatible maps [7] in probabilistic settings and proved a number of fixed point theorems in this direction. Several interesting and elegant results have been obtained by various authors on different settings (see [1–9, 12–14, 17, 18, 25]).

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In the present paper, we prove common fixed point theorems for occasionally weakly compatible self maps in Menger spaces. Our results never require the completeness of the whole space, continuity of the involved maps and containment of ranges amongst involved maps.

2. Preliminaries

Definition 2.1. [20] A triangular norm $*$ (shortly t-norm) is a binary operation on the unit interval $[0,1]$ such that for all $a,b,c,d \in [0,1]$ the following conditions are satisfied:

(1) $a * 1 = a$;

(2) $a * b = b * a$;

(3) $a * b \leq c * d$, whenever $a \leq c$ and $b \leq d$;

(4) $a * (b * c) = (a * b) * c$.

The following are the basic t-norms:

$$a * b = \min\{a, b\};$$
$$a * b = a.b;$$
$$a * b = \max\{a + b - 1, 0\}.$$

Definition 2.2. [20] A mapping $F : \mathbb{R} \to [0,\infty)$ is said to be a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$.

We shall denote by $\Im$ the set of all distribution functions while H will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \leq 0; \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a non-empty set, $\mathcal{F} : X \times X \to \Im$ is called a probabilistic distance on X and the value of $\mathcal{F}$ at $(x,y) \in X \times X$ is represented by $F_{x,y}$.

Definition 2.3. [20] The ordered pair $(X, \mathcal{F})$ is called a PM-space if X is a non-empty set and F is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,

(1) $F_{x,y} = H$ iff $x = y$;

(2) $F_{x,y} = F_{y,x}$;

(3) If $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$ then $F_{x,z}(t+s) = 1$. The ordered triple $(X, \mathcal{F}, *)$ is called a Menger space if $(X, \mathcal{F})$ is a PM-space, $*$ is a t-norm and the following inequality holds:

$$F_{x,z}(t + s) \geq F_{x,y}(t) * F_{y,z}(s)$$

for all $x, y, z \in X$ and $t, s > 0$.

Every metric space (X, d) can always be realized as a PM-space by considering $F : X \times X \to \Im$ defined by $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$. So PM-spaces offer a wider framework than that of metric spaces and are better suited to cover even wider statistical situations.

Definition 2.4. [20] Let $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $*$.

(1) A sequence $\{x_n\}$ in X is said to be converge to a point x in X if and only if for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N(\epsilon, \lambda)$ such that $F_{x_n,x}(\epsilon) > 1 - \lambda$ for all $n \geq N(\epsilon, \lambda)$.

(2) A sequence $\{x_n\}$ in X is said to be Cauchy if for every $\epsilon > 0$ and $\lambda > 0$, there exists a positive integer $N(\epsilon, \lambda)$ such that $F_{x_n,x_m}(\epsilon) > 1 - \lambda$ for all $n, m \geq N(\epsilon, \lambda)$.

(3) A Menger space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.5. [16] Self maps A and B of a Menger space $(X, \mathcal{F}, *)$ are said to be compatible if $F_{ABx_n,BAx_n}(t) \to 1$ for all $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $Ax_n, Bx_n \to z$ for some z in X as $n \to \infty$.

Lemma 2.1. [16] *Let $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $*$. If there exists a constant $k \in (0, 1)$ such that $F_{x,y}(kt) \geq F_{x,y}(t)$ for all $x, y \in X$ and $t > 0$ then $x = y$.*

Definition 2.6. [23] Self maps A and B of a non-empty set X are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

Remark 2.1. [23] If self maps A and B of a non-empty set X are compatible then they are weakly compatible.

Definition 2.7. [12] Self maps A and B of a non-empty set X are occasionally weakly compatible if and only if there is a point $x \in X$ which is a coincidence point of A and B at which A and B commute.

From the following example, it is clear that the notion of occasionally weakly compatible maps is more general than weak compatibility.

Example 2.1. Let $X = [0, \infty)$ with the usual metric. Define $A, B : X \to X$ by $Ax = 3x$ and $Bx = x^2$ for all $x \in X$. Then $Ax = Bx$ for $x = 0, 3$ but $AB(0) = BA(0)$, and $AB(3) \neq BA(3)$. Thus A and B are occasionally weakly compatible maps but not weakly compatible.

Lemma 2.2. [12] *Let X be a non-empty set, A and B are occasionally weakly compatible self maps of X. If A and B have a unique point of coincidence, $w = Ax = Bx$, then w is the unique common fixed point of A and B.*

3. Results

THEOREM 3.1. *Let* $(X, \mathcal{F}, *)$ *be a Menger space with continuous t-norm* $* = \min$. *Further, let the pairs* (A, S) *and* (B, T) *are occasionally weakly compatible in* X *satisfying*

$$[1 + aF_{Sx,Ty}(kt)] * F_{Ax,By}(kt) \geq a \min \left\{ \begin{array}{c} F_{Ax,Sx}(kt) * F_{By,Ty}(kt), \\ F_{Ax,Ty}(2kt) * F_{By,Sx}(2kt) \end{array} \right\} + \left\{ \begin{array}{c} F_{Sx,Ty}(t) * F_{Ax,Sx}(t) * F_{By,Ty}(t) \\ *F_{Ax,Ty}(2t) * F_{By,Sx}(2t) \end{array} \right\} \tag{1}$$

for all $t > 0$ *and* $x, y \in X$ *with fixed constants* $a \in (-1, 0]$ *and* $k \in (0, 1)$. *Then there exists a unique point* $w \in X$ *such that* $Aw = Sw = w$ *and a unique point* $z \in X$ *such that* $Bz = Tz = z$. *Moreover,* $z = w$, *so that there is a unique common fixed point of* A, B, S *and* T.

Proof. Since the pairs (A, S) and (B, T) are each occasionally weakly compatible, there exist points $x, y \in X$ such that $Ax = Sx$, $ASx = SAx$ and $By = Ty$, $BTy = TBy$. Now we show that $Ax = By$. By inequality (1), we get

$$\begin{aligned} &[1 + aF_{Ax,By}(kt)] * F_{Ax,By}(kt) \\ &\geq a \min \left\{ \begin{array}{c} F_{Ax,Ax}(kt) * F_{By,By}(kt), \\ F_{Ax,By}(2kt) * F_{By,Ax}(2kt) \end{array} \right\} + \left\{ \begin{array}{c} F_{Ax,By}(t) * F_{Ax,Ax}(t) * F_{By,By}(t) \\ *F_{Ax,By}(2t) * F_{By,Ax}(2t) \end{array} \right\} \\ &\qquad \geq a \min \left\{ \begin{array}{c} F_{Ax,Ax}(kt) * F_{By,By}(kt), F_{Ax,Ax}(kt) \\ *F_{Ax,By}(kt) * F_{By,Ax}(kt) * F_{Ax,Ax}(kt) \end{array} \right\} \\ &\qquad + \left\{ \begin{array}{c} F_{Ax,By}(t) * F_{Ax,Ax}(t) * F_{By,By}(t) \\ *F_{Ax,Ax}(t) * F_{Ax,By}(t) * F_{By,Ax}(t) * F_{Ax,Ax}(t) \end{array} \right\} \\ &= a \min \{1 * 1, 1 * F_{Ax,By}(kt) * F_{By,Ax}(kt) * 1\} + \left\{ \begin{array}{c} F_{Ax,By}(t) * 1 * 1 * 1 \\ *F_{Ax,By}(t) * F_{By,Ax}(t) * 1 \end{array} \right\}, \end{aligned}$$

after simplification, we have

$$\begin{aligned} F_{Ax,By}(kt) + aF_{Ax,By}(kt) &\geq aF_{Ax,By}(kt) + F_{Ax,By}(t) \\ F_{Ax,By}(kt) &\geq F_{Ax,By}(t). \end{aligned}$$

Thus, by Lemma 2.1, $Ax = By$. Therefore, $Ax = Sx = By = Ty$. Moreover, if there is another point z such that $Az = Sz$. Then using inequality (1) it follows that $Az = Sz = By = Ty$, or $Ax = Az$. Hence $w = Ax = Sx$ is the unique point of coincidence of A and S. By Lemma 2.2, w is the unique common fixed point of A and S. Similarly, there is a unique point $z \in X$ such that $z = Bz = Tz$. Suppose that $w \neq z$ and using inequality (1), we get

$$\begin{aligned} &[1 + aF_{Sw,Tz}(kt)] * F_{Aw,Bz}(kt) \\ &\geq a \min \left\{ \begin{array}{c} F_{Aw,Sw}(kt) * F_{Bz,Tz}(kt), \\ F_{Aw,Tz}(2kt) * F_{Bz,Sw}(2kt) \end{array} \right\} + \left\{ \begin{array}{c} F_{Sw,Tz}(t) * F_{Aw,Sw}(t) * F_{Bz,Tz}(t) \\ *F_{Aw,Tz}(2t) * F_{Bz,Sw}(2t) \end{array} \right\} \end{aligned}$$

$$= a \min \left\{ \begin{array}{c} F_{w,w}(kt) * F_{z,z}(kt), \\ F_{w,z}(2kt) * F_{z,w}(2kt) \end{array} \right\} + \left\{ \begin{array}{c} F_{w,z}(t) * F_{w,w}(t) * F_{z,z}(t) \\ *F_{w,z}(2t) * F_{z,w}(2t) \end{array} \right\}$$

$$[1 + aF_{Sw,Tz}(kt)] * F_{Aw,Bz}(kt) \geq$$

$$a \min \left\{ \begin{array}{c} F_{w,w}(kt) * F_{z,z}(kt), F_{w,z}(kt) \\ *F_{z,z}(kt) * F_{z,z}(kt) * F_{z,w}(kt) \end{array} \right\} + \left\{ \begin{array}{c} F_{w,z}(t) * F_{w,w}(t) * F_{z,z}(t) * F_{w,z}(t) \\ *F_{z,z}(t) * F_{z,z}(t) * F_{z,w}(t) \end{array} \right\}$$

$$= a \min \{1 * 1, F_{w,z}(kt) * 1 * 1 * F_{z,w}(kt)\} + \left\{ \begin{array}{c} F_{w,z}(t) * 1 * 1 * F_{w,z}(t) \\ *1 * 1 * F_{z,w}(t) \end{array} \right\},$$

after simplification, we have

$$F_{w,z}(kt) + aF_{w,z}(kt) \geq aF_{w,z}(kt) + F_{w,z}(t)$$
$$F_{w,z}(kt) \geq F_{w,z}(t).$$

Thus by Lemma 2.1, $w = z$. Therefore w is the unique common fixed point of A, B, S and T in X. ∎

From Theorem 3.1 with $a = 0$, we have the following result:

COROLLARY 3.1. *Let $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $* = \min$. Further, let the pairs (A, S) and (B, T) are occasionally weakly compatible in X satisfying*

$$F_{Ax,By}(kt) \geq \left\{ \begin{array}{c} F_{Sx,Ty}(t) * F_{Ax,Sx}(t) * F_{By,Ty}(t) \\ *F_{Ax,Ty}(2t) * F_{By,Sx}(2t) \end{array} \right\} \quad (2)$$

for all $t > 0$ and $x, y \in X$ with fixed constant $k \in (0, 1)$. Then there exists a unique point $w \in X$ such that $Aw = Sw = w$ and a unique point $z \in X$ such that $Bz = Tz = z$. Moreover, $z = w$, so that there is a unique common fixed point of A, B, S and T.

EXAMPLE 3.1. Let $X = [0, 4]$ with the metric d defined by $d(x, y) = |x - y|$ and for each $t \in [0, 1]$, define

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

for all $x, y \in X$. Clearly $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $* = \min$. Define the self maps A, B, S and T defined by

$$A(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 2; \\ 4, & \text{if } 2 < x \leq 4. \end{cases} \quad S(x) = \begin{cases} 2, & \text{if } 0 \leq x \leq 2; \\ 0, & \text{if } 2 < x \leq 4. \end{cases}$$
$$B(x) = \begin{cases} 2, & \text{if } 0 \leq x \leq 2; \\ 4, & \text{if } 2 < x \leq 4. \end{cases} \quad T(x) = \begin{cases} 2, & \text{if } 0 \leq x \leq 2; \\ \frac{x}{4}, & \text{if } 2 < x \leq 4. \end{cases}$$

Then A, B, S and T satisfy all the conditions of Theorem 3.1 and Corollary 3.1 for some $k \in (0, 1)$. That is, $A(2) = 2 = S(2), AS(2) = 2 = SA(2)$ and $B(2) = 2 =$

$T(2)$, $BT(2) = 2 = TB(2)$. Also A and B as well as S and T are occasionally weakly compatible maps. Hence, 2 is the unique common fixed point of A, B, S and T. This example never requires any condition on the containment of ranges of the involved maps. Further, self maps A, B, S and T are discontinuous at 2.

On taking $A = B$ and $S = T$ in Theorem 3.1 then we get the interesting result.

THEOREM 3.2 *Let $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $* =$ min. Further, let the pair (A, S) is occasionally weakly compatible in X satisfying*

$$[1 + aF_{Sx,Sy}(kt)] * F_{Ax,Ay}(kt) \geq a \min \left\{ \begin{matrix} F_{Ax,Sx}(kt) * F_{Ay,Sy}(kt), \\ F_{Ax,Sy}(2kt) * F_{Ay,Sx}(2kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{Sx,Sy}(t) * F_{Ax,Sx}(t) \\ *F_{Ay,Sy}(t) * F_{Ax,Sy}(2t) * F_{Ay,Sx}(2t) \end{matrix} \right\} \tag{3}$$

for all $t > 0$ and $x, y \in X$ with fixed constants $a \in (-1, 0]$ and $k \in (0, 1)$. Then A and S have a unique common fixed point in X.

Proof. Since A and S are occasionally weakly compatible, there exists a point $u \in X$ such that $Au = Su, ASu = SAu$. Now we show that Au is the unique common fixed point of A and S. Then from inequality (3), we get

$$[1 + aF_{Su,SAu}(kt)] * F_{Au,AAu}(kt) \geq a \min \left\{ \begin{matrix} F_{Au,Su}(kt) * F_{AAu,SAu}(kt), \\ F_{Au,SAu}(2kt) * F_{AAu,Su}(2kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{Su,SAu}(t) * F_{Au,Su}(t) \\ *F_{AAu,SAu}(t) * F_{Au,SAu}(2t) \\ *F_{AAu,Su}(2t) \end{matrix} \right\}$$

$$\begin{aligned} [1 + aF_{Au,AAu}(kt)] * F_{Au,AAu}(kt) \geq\; & a \min \left\{ \begin{matrix} F_{Au,Au}(kt) * F_{AAu,AAu}(kt), \\ F_{Au,AAu}(2kt) * F_{AAu,Au}(2kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{Au,AAu}(t) * F_{Au,Au}(t) \\ *F_{AAu,AAu}(t) * F_{Au,AAu}(2t) \\ *F_{AAu,Au}(2t) \end{matrix} \right\} \\ \geq\; & a \min \left\{ \begin{matrix} F_{Au,Au}(kt) * F_{AAu,AAu}(kt), \\ F_{Au,Au}(kt) * F_{Au,AAu}(kt) * F_{AAu,Au}(kt) * F_{Au,Au}(kt) \end{matrix} \right\} \\ & + \left\{ \begin{matrix} F_{Au,AAu}(t) * F_{Au,Au}(t) * F_{AAu,AAu}(t) \\ *F_{Au,Au}(t) * F_{Au,AAu}(t) * F_{AAu,Au}(t) * F_{Au,Au}(t) \end{matrix} \right\} \\ =\; & a \min \left\{ \begin{matrix} 1 * 1, 1 * F_{Au,AAu}(kt) \\ *F_{AAu,Au}(kt) * 1 \end{matrix} \right\} + \left\{ \begin{matrix} F_{Au,AAu}(t) * 1 * 1 * 1 \\ *F_{Au,AAu}(t) * F_{AAu,Au}(t) * 1 \end{matrix} \right\}. \end{aligned}$$

After simplification, we have

$$F_{Au,AAu}(kt) + aF_{Au,AAu}(kt) \geq aF_{Au,AAu}(kt) + F_{Au,AAu}(t)$$
$$F_{Au,AAu}(kt) \geq F_{Au,AAu}(t).$$

Thus by Lemma 2.1, $Au = AAu$. Hence $AAu = ASu = SAu = Au$. So, it is clear that Au is a point of coincidence of A and S that is $AAu = SAu = Au$, and $w = Au$ is a common fixed point of A and S.

Uniqueness: Suppose that $v(v \neq w)$ be another common fixed point of the self maps A and S. Then from inequality (3), we get

$$[1 + aF_{Sw,Sv}(kt)] * F_{Aw,Av}(kt) \geq$$
$$a \min \left\{ \begin{matrix} F_{Aw,Sw}(kt) * F_{Av,Sv}(kt), \\ F_{Aw,Sv}(2kt) * F_{Av,Sw}(2kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{Sw,Sv}(t) * F_{Aw,Sw}(t) * F_{Av,Sv}(t) \\ *F_{Aw,Sv}(2t) * F_{Av,Sw}(2t) \end{matrix} \right\}$$

$$[1 + aF_{w,v}(kt)] * F_{w,v}(kt) \geq$$
$$a \min \left\{ \begin{matrix} F_{w,w}(kt) * F_{v,v}(kt), \\ F_{w,v}(2kt) * F_{v,w}(2kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{w,v}(t) * F_{w,w}(t) * F_{v,v}(t) \\ *F_{w,v}(2t) * F_{v,w}(2t) \end{matrix} \right\}$$
$$\geq a \min \left\{ \begin{matrix} F_{w,w}(kt) * F_{v,v}(kt), F_{w,v}(kt) \\ *F_{v,v}(kt) * F_{v,v}(kt) * F_{v,w}(kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{w,v}(t) * F_{w,w}(t) * F_{v,v}(t) * F_{w,v}(t) \\ *F_{v,v}(t) * F_{v,v}(t) * F_{v,w}(t) \end{matrix} \right\}$$
$$= a \min \left\{ \begin{matrix} 1 * 1, F_{w,v}(kt) * 1 \\ *1 * F_{v,w}(kt) \end{matrix} \right\} + \left\{ \begin{matrix} F_{w,v}(t) * 1 * 1 \\ *F_{w,v}(t) * 1 * 1 * F_{v,w}(t) \end{matrix} \right\}$$

after simplification, we have

$$F_{w,v}(kt) + aF_{w,v}(kt) \geq aF_{w,v}(kt) + F_{w,v}(t)$$
$$F_{w,v}(kt) \geq F_{w,v}(t).$$

Thus by Lemma 2.1, $w = v$. That is w is the unique common fixed point of A and S in X. ■

From Theorem 3.2 with $a = 0$, we have the following:

Corollary 3.2. *Let $(X, \mathcal{F}, *)$ be a Menger space with continuous t-norm $* =$ min. Further, let the pair (A, S) is occasionally weakly compatible in X satisfying*

$$F_{Ax,Ay}(kt) \geq \left\{ \begin{matrix} F_{Sx,Sy}(t) * F_{Ax,Sx}(t) * F_{Ay,Sy}(t) \\ *F_{Ax,Sy}(2t) * F_{Ay,Sx}(2t) \end{matrix} \right\} \tag{4}$$

for all $t > 0$ and $x, y \in X$ with fixed constant $k \in (0, 1)$. Then A and S have a unique common fixed point in X.

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