MATEMATIČKI VESNIK
МАТЕМАТИЧКИ ВЕСНИК
75, 4 (2023), 247–264
December 2023

research paper
оригинални научни рад
DOI: 10.57016/MV-ZUXV3844

EQUIPRIME FUZZY GRAPH OF A NEARRING WITH RESPECT TO A LEVEL IDEAL

B. Jagadeesha, Babushri Srinivas Kedukodi and Syam Prasad Kuncham

Abstract. In this paper, we introduce an equiprime fuzzy graph of a nearring with respect to the level ideal of a fuzzy ideal. We interrelate graph theoretical properties of the graph and ideal theoretical properties of nearring. We show that the properties like vertex cut, connectedness of the graph depend on the properties of the fuzzy ideal. We define ideal symmetry of the graph and find conditions for the graph to be ideal symmetric. If the fuzzy ideal is equiprime then we show that the level set induces a fuzzy clique. We find conditions required for the level set to be the vertex cover of the graph. We find interrelation between equiprime fuzzy graph and fuzzy graph of nearring with respect to level ideal. We study properties of the graph under neaaring homomorphism. We prove that the connectedness of the graph in homomorphic image depends on properties of ideal. We obtain conditions required for homomorphic image of an equiprime fuzzy ideal to be an equiprime fuzzy ideal.

1. Introduction

Algebra and graph theory are independent areas of mathematics, and researchers have attempted to link these areas. The study of the relation between algebraic structures and graph theory was initiated by Beck [6] by introducing the zero divisor graph of a commutative ring. Redmond [25] generalized the zero divisor graph using an ideal and defined an ideal-based zero divisor graph of a commutative ring. Bhavanari, Kuncham and Kedukodi [8] proposed the graph of a generalized ring (nearring) with respect to an ideal and introduced a new type of symmetry of the graph called ideal symmetry. Pang, Zhang, Zhang and Wang [23] studied the problem of dominating set in directed graphs.

Fuzzy set theory is applied in various areas of mathematics such as algebra, graph theory, topology, and differential equations to generalize various constraints in these areas. Bhakat and Das [7] used fuzzy set theory to generalize the definition of the ideal and introduced the fuzzy ideal of a ring. Davvaz [10] generalized fuzzy ideals and

2020 Mathematics Subject Classification: 16Y30, 03E72, 16Y99
Keywords and phrases: Nearring; equiprime; ideal; fuzzy graph; homomorphism.

introduced threshold-based fuzzy ideals. Kedukodi, Kuncham, and Bhavanari [18] studied threshold-based equiprime, 3-prime, and c-prime fuzzy ideals of nearrings and used them to study roughness in rings. Akram [2] generalized the definition of fuzzy ideals using triangular norms by introducing T-fuzzy ideals of nearrings and studied Artinian and Northenian nearrings. Shum and Akram [26] determined properties of intuitionistic (T,S)-fuzzy ideals of a nearring and found their properties. Lattice is an algebraic structure useful for studying order between different sets and between elements of a set. To study ordered fuzzy sets, Goguen [11] introduced lattice valued fuzzy (L-fuzzy sets). Interval valued fuzzy sets are another generalization of fuzzy set theory where the fuzzy membership function is represented by a closed subinterval of $[0,1]$. Davvaz [9] used interval-valued fuzzy sets and L-fuzzy sets to begin the study of interval-valued L-fuzzy ideals of nearrings. Jagadeesha, Kedukodi, and Kuncham [15, 16, 19] studied various types of fuzzy ideals of nearrings using triangular norms and interval-valued L-fuzzy sets.

Researchers used fuzzy set theory to generalize graph theory. Mordeson and Peng [22] introduced basic operations such as Cartesian product, union, intersection on fuzzy subgraphs. Koczy [20] introduced the concept of vertex fuzzy graph and applied it to distributed communication switching systems. Akram and Nawaz [4] related soft set theory and fuzzy graph theory by introducing fuzzy soft graphs. Akram and Dudek [3] related interval-valued fuzzy set theory and graph theory by introducing interval-valued fuzzy graphs. Akram [1] introduced a new type of fuzzy graphs, namely m-polar fuzzy graphs, and applied them to image processing and decision making.

In this paper, we relate equiprime fuzzy ideal of a nearring with a fuzzy graph. We introduce equiprime fuzzy graph of nearring N with respect to the level ideal ν_t of a fuzzy ideal ν denoted by (N_e, ν, ρ_e, t). We study the properties of the fuzzy graph. If x is an element of the fuzzy ideal, then we prove that x is connected to all other vertices of the fuzzy graph. If the fuzzy ideal is an equiprime fuzzy ideal, then the level set ν_t is a strong vertex cut of the fuzzy graph. We define the ideal symmetry of the graph and prove that if ν is an equiprime fuzzy ideal, then the fuzzy graph is ideal symmetric. We find conditions for a fuzzy clique of the fuzzy graph. If ν is an equiprime fuzzy ideal, then we prove that ν_t is the vertex cover of the fuzzy graph. If nearring is a simple nearring and the fuzzy ideal is an equiprime fuzzy ideal, then we prove that the fuzzy graph is a complete subgraph or a star graph. We find conditions under which the fuzzy graphs of nearring, c-prime fuzzy graph and equiprime fuzzy graph defined by Kedukodi, Kuncham and Bhavanari [17] are equivalent. If a fuzzy ideal is c-prime, then we prove that the fuzzy graph and the c-prime fuzzy graph are identical. If N_1 and N_2 are nearrings and f is a nearring homomorphism between N_1, N_2, then f is a graph homomorphism between (N_{1e}, ν, ρ_e, t) and $(N_{2e}, f(\nu), f(\rho)_e, t)$. We prove that if x is connected to all vertices of (N_{1e}, ν, ρ_e, t), then $f(x)$ is connected to all other vertices of $(N_{2e}, f(\nu), f(\rho)_e, t)$. We find conditions necessary for a homomorphic image of an equiprime fuzzy ideal to be an equiprime fuzzy ideal.

2. Preliminaries

In this paper N, N_1 and N_2 represent right nearrings. We refer to Pilz [24] for definitions, concepts related to nearrings, Anderson and Fuller [5] for rings, Harary [13] for graphs.

Definition 2.1 ([14]). Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. A *graph homomorphism* from G_1 to G_2 is a mapping $f : V_1 \to V_2$ such that $(f(u), f(v)) \in E_2$ whenever $(u, v) \in E_1$.

Definition 2.2 ([21]). A fuzzy graph $H = (\nu, \rho)$ of (V, E) is defined by a fuzzy subset ν of V and a fuzzy subset ρ of E such that $\rho(x, y) \leq \nu(x) \wedge \nu(y)\ \forall\ x, y \in V$.

Definition 2.3 ([21]). Let $t \in [0, 1]$. Then $\nu_t = \{x \in V \mid \nu(x) \geq t\}$ is called the level set of ν. Also $\rho_t = \{(x, y) \in E \mid \rho(x, y) \geq t\}$ is called the level set of ρ. Then (ν_t, ρ_t) is a graph with vertex set ν_t and edge set ρ_t. Let (ν, ρ) be a fuzzy graph. Then (ν, ρ) is said to be *complete* if $\rho(u, v) = \nu(u) \wedge \nu(v)\ \forall\ u, v \in V$. A complete fuzzy subgraph with maximum number of vertices is called a *fuzzy clique* of (ν, ρ).

Definition 2.4 ([24]). Let I, J, K be ideals of N. Then I is said to be a *prime ideal* if $JK \subseteq I$ implies $J \subseteq I$ or $K \subseteq I$. Nearring N is said to be *integral* if it has no zero divisors. Nearring N is said to be a *zerosymmetric* if $x0 = 0$ for all $x \in N$. An ideal I of N is said to be *totally reflexive* if $aNb \subseteq I$ then $bNa \subseteq I$ for all $a, b \in N$. Nearring N is said to be *simple* if its only ideals are $\{0\}$ and N. Let N_1, N_2 be nearrings. Then $g : N_1 \to N_2$ is said to be a *nearring homomorphism* if for all $a, b \in N_1$, (i) $g(a + b) = g(a) + g(b)$, (ii) $g(ab) = g(a)g(b)$.

Definition 2.5 ([12]). Let I be an ideal of N. Then I is said to be a *c-prime ideal* of N if $xy \in I$ implies $x \in I$ or $y \in I$ for all $x, y \in N$. I is said to be an *equiprime ideal* if $a, x, y \in N$ with $anx - any \in I$ for all $n \in N$ implies $a \in I$ or $x - y \in I$. I is said to be a *3-prime ideal* if $x, y \in N$ and $xny \in I$ for all $n \in N$ implies $x \in I$ or $y \in I$.

Theorem 2.6 ([12, 27]). *The following implications hold in nearrings:*
(i) each equiprime ideal is a 3-prime ideal;

(ii) each c-prime ideal is a 3-prime ideal.
The notions of equiprime ideal, 3-prime ideal and prime ideal coincide in rings. In commutative rings, all the above notions coincide.

Definition 2.7 ([12]). An ideal I of N is said to be a *c-semiprime ideal* if $x \in N$ and $x^2 \in I$ implies $x \in I$. An ideal I of N is said to be an *equisemiprime ideal* if, for $a, b \in N$, $(a - b)ra - (a - b)rb \in I$ for all $r \in N$ implies $(a - b) \in I$.

Definition 2.8 ([10]). Let $\alpha, \beta \in [0, 1]$ and $\alpha < \beta$. Let ν be a fuzzy subset of N. Then ν is called a fuzzy ideal with thresholds α, β if for all $x, y, i \in N$,
(i) $\alpha \vee \nu(x + y) \geq \beta \wedge \nu(x) \wedge \nu(y)$, (ii) $\alpha \vee \nu(-x) \geq \beta \wedge \nu(x)$,

(iii) $\alpha \vee \nu(y + x - y) \geq \beta \wedge \nu(x)$, (iv) $\alpha \vee \nu(xy) \geq \beta \wedge \nu(x)$,

(v) $\alpha \vee \nu(x(y + i) - xy) \geq \beta \wedge \nu(i)$.

Here α is called the lower threshold of ν and β as the upper threshold of ν. In this paper fuzzy ideal ν means fuzzy ideal with lower threshold α and upper threshold β.

THEOREM 2.9 ([10]). *Let ν be a fuzzy subset of N. Then ν is a fuzzy ideal of N if and only if for every $t \in (\alpha, \beta]$ the level subset ν_t is an ideal of N.*

DEFINITION 2.10 ([18]). Let α, $\beta \in [0,1]$ and $\alpha < \beta$. Let ν be a fuzzy ideal with thresholds α, β. If $x, y, a, b \in N$ then
(i) ν is called an *equiprime fuzzy ideal with thresholds* α, β if $\alpha \vee \nu(a) \vee \nu(x-y) \geq \beta \wedge \inf_{r\in N} \nu(arx - ary)$.

(ii) ν is called a *3-prime fuzzy ideal with thresholds* α, β if $\alpha \vee \nu(a) \vee \nu(b) \geq \beta \wedge \inf_{r\in N} \nu(arb)$.

(iii) μ is called a *c-prime fuzzy ideal with thresholds* α, β if $\alpha \vee \nu(a) \vee \nu(b) \geq \beta \wedge \nu(ab)$.

(iv) A fuzzy ideal μ is called an *equisemiprime fuzzy ideal* if for all $a \in N$, $\alpha \vee \nu(a-b) \geq \beta \wedge \inf_{r\in N} \nu((a-b)ra - (a-b)rb)$.

THEOREM 2.11 ([18]). *Let ν be a fuzzy ideal of N. Then ν is an equiprime (resp. equisemiprime) fuzzy ideal of N if and only if for every $t \in (\alpha, \beta]$, the level subset ν_t is a equiprime (resp. equisemiprime) ideal of N.*

DEFINITION 2.12 ([9]). Let $f : N_1 \to N_2$ be a mapping. Let μ be a fuzzy subset of N_1. Then the image of μ under f is given by

$$f(\hat{\mu})(y) = \begin{cases} \vee_{x\in f^{-1}(y)} \hat{\mu}(x) & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

for all $y \in N_2$.

DEFINITION 2.13 ([17]). Let $\nu : N \to (0,1]$ be a fuzzy ideal of N. Let $t \in (\alpha, \beta]$ be fixed. Define $\rho : N \times N \to [0,1]$ as follows:

$$\rho_c(x,y) = \begin{cases} \nu(x) \wedge \nu(y) & x \neq y \text{ and } (xNy \subseteq \nu_t \text{ or } yNx \subseteq \nu_t) \\ 0 & \text{otherwise.} \end{cases}$$

Then the fuzzy graph with respect to ν_t denoted by (N_3, ν, ρ_3, t) is called *fuzzy graph of N with respect to level ideal ν_t.*

DEFINITION 2.14. Let $\nu : N \to (0,1]$ be a fuzzy ideal of N. Let $t \in (\alpha, \beta]$ be fixed. Define $\rho : N \times N \to [0,1]$ as follows:

$$\rho_c(x,y) = \begin{cases} \nu(x) \wedge \nu(y) & x \neq y \text{ and } (xy \in \nu_t \text{ or } yx \in \nu_t) \\ 0 & \text{otherwise} \end{cases}$$

Then the fuzzy graph with respect to ν_t denoted by (N_c, ν, ρ_c, t) is called *c-prime fuzzy graph of N with respect to level ideal ν_t.*

REMARK 2.15. In the next part of this paper by fuzzy (resp. fuzzy equiprime, fuzzy equisemiprime) ideal we mean fuzzy (resp. fuzzy equiprime, fuzzy equisemiprime) ideal with lower threshold α upper threshold β.

3. Equiprime fuzzy graph of nearring with respect to level ideal

Definition 3.1. Let $\nu : N \to (0,1]$ be a fuzzy ideal of N. Let $t \in (\alpha, \beta]$ be fixed. For $p \in N$, define $\rho_p : N \times N \to [0,1]$ as follows:

$$\rho_p(p,x) = \begin{cases} \nu(p) \wedge \nu(x) & \text{if } p \neq x \text{ and } (prx - pr0 \in \nu_t) \\ & \text{or } (xrp - xr0 \in \nu_t) \text{ for all } r \in N \\ 0 & \text{otherwise.} \end{cases}$$

Then the fuzzy graph (N, ν, ρ_p) is called the fuzzy graph of N with respect to p and level ideal ν_t. We denote this fuzzy graph by (N_e, ν, ρ_p, t).

Let $(N, \nu, \rho) = \bigcup_{p \in N} (N, \nu, \rho_p)$. Then the fuzzy graph (N, ν, ρ) is called the *equiprime fuzzy graph of N with respect level ideal ν_t*. We denote this fuzzy graph by (N_e, ν, ρ_e, t)

Now we provide examples for equiprime fuzzy graph of nearring with respect to level ideal.

Example 3.2. Let $N = \{0, a, b, c\}$ be the nearring with addition and multiplication defined as in Table 1.

$+$	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

$\cdot$	0	a	b	c
0	0	0	0	0
a	0	a	0	a
b	b	b	b	b
c	b	c	b	c

Table 1: Nearring for Example 3.2

We define $\nu : N \to (0,1]$ by

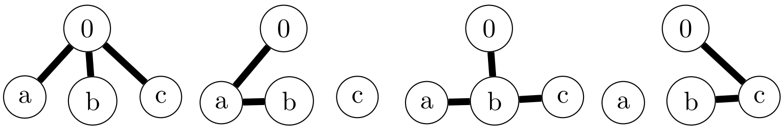
$$\nu(x) = \begin{cases} 0.8 & \text{if } x = 0 \\ 0.5 & \text{if } x = b \\ 0.1 & \text{if } x \in \{a, c\}. \end{cases}$$

If we take thresholds $\alpha = 0.1$ and $\beta = 0.5$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0, b\}$ and the values of $\rho_p(p,x)$ are given in Table 2.

$\rho_p(p,x)$	$x = 0$	$x = a$	$x = b$	$x = c$
$\rho_0(0,x)$	0	0.1	0.5	0.1
$\rho_a(a,x)$	0.1	0	0.1	0
$\rho_b(b,x)$	0.5	0.1	0	0.1
$\rho_c(c,x)$	0.1	0	0.1	0

Table 2: $\rho_p(p,x)$ when $\nu_t = \{0, b\}$

Graphs $(N_e, \nu, \rho_0, t), (N_e, \nu, \rho_a, t), (N_e, \nu, \rho_b, t), (N_e, \nu, \rho_c, t)$ are shown in Figures 1a, 1b, 1c and 1d respectively and the graph (N_e, ν, ρ_e, t) is shown on Figure 2.



(a) (N_e, ν, ρ_0, t) (b) (N_e, ν, ρ_a, t) (c) (N_e, ν, ρ_b, t) (d) (N_e, ν, ρ_c, t)

Figure 1: $EQ_I^p(N)$

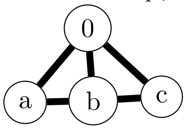


Figure 2: (N_e, ν, ρ_e, t)

EXAMPLE 3.3. Let $N = Z_8 = \{0, 1, 2, 3, 4, 5, 6, 7\}$ be the ring of integers modulo 8. We define

$$\nu(x) = \begin{cases} 0.9 & \text{if } x = 0, \\ 0.7 & \text{if } x = 4, \\ 0.3 & \text{if } x \in \{2, 6\}, \\ 0.1 & \text{otherwise.} \end{cases}$$

If we take thresholds $\alpha = 0.1$ and $\beta = 0.3$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0, 2, 4, 6\}$ and the values of $\rho_p(p, x)$ are given in Table 3. Graph (N_e, ν, ρ_e, t) is shown on Figure 3.

$\rho_p(p, x)$	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$	$x = 6$	$x = 7$
$\rho_0(0, x)$	0	0.1	0.3	0.1	0.7	0.1	0.3	0.1
$\rho_1(1, x)$	0.1	0	0.1	0	0.1	0	0.1	0
$\rho_2(2, x)$	0.3	0.1	0	0.1	0.3	0.1	0.3	0.1
$\rho_3(3, x)$	0.1	0	0.1	0	0.1	0	0.1	0
$\rho_4(4, x)$	0.7	0.1	0.3	0.1	0	0.1	0.3	0.1
$\rho_5(5, x)$	0.1	0	0.1	0	0.1	0	0.1	0
$\rho_6(6, x)$	0.3	0.1	0	0.1	0.3	0.1	0.3	0.1
$\rho_7(7, x)$	0.1	0	0.1	0	0.1	0	0.1	0

Table 3: $\rho_p(p, x)$ when $\nu_t = \{0, 2, 4, 6\}$

PROPOSITION 3.4. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$.*
(i) If $p \in \nu_t$ then (N_e, ν, ρ_p, t) is a star graph with root vertex p.

(ii) If ν is an equiprime fuzzy ideal of N and (N_e, ν, ρ_p, t) is a star graph with root vertex p then $p \in \nu_t$.

(iii) Let $x \in N$. If $x \in \nu_t$ then x is connected to all other vertices of (N_e, ν, ρ_e, t).

Proof. Let $t \in (\alpha, \beta]$. Then ν_t is an ideal of N. To prove (i), let $p \in \nu_t$ and $x \in N$. Let $r \in N$ be arbitrarily fixed. By the property of ideal, $prx \in \nu_t$ and $pr0 \in \nu_t$. Then $prx - pr0 \in \nu_t$ (by property of ideal.) Hence p is adjacent to x in (N_e, ν, ρ_p, t).

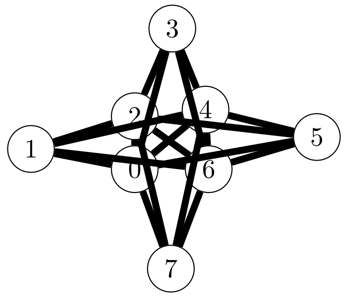


Figure 3: (N_e, ν, ρ_e, t) when $\nu_t = \{0, 2, 4, 6\}$

Therefore p is adjacent to all other vertices of (N_e, ν, ρ_p, t). Thus (N_e, ν, ρ_p, t) is a star graph with root vertex p.

To prove (ii), let (N_e, ν, ρ_p, t) be a star graph with root vertex p. Then by definition for any x in N, $prx - pr0 \in \nu_t$ for all $r \in N$ or $xrp - xr0 \in \nu_t$ for all $r \in N$. Without loss of generality assume $prx - pr0 \in I$ for all $r \in N$. Suppose $\nu_t = N$. Then $p \in \nu_t$. Suppose $\nu_t \subset N$. Choose $x \in N \setminus \nu_t$. By the property of equiprime fuzzy ideal of N, we get ν_t is an equiprime ideal of N. Then we get $p \in \nu_t$.

To prove (iii), let $x \in \nu_t$. Then by (i), (N_e, ν, ρ_x, t) is a star graph with root vertex x. Then x is adjacent with all other vertices of N in (N_e, ν, ρ_x, t). By definition of equiprime fuzzy graph we get x is adjacent to all other vertices of (N_e, ν, ρ_e, t). □

Now we provide examples to show the conditions in Proposition 3.4 are necessary.
(i) In Example 3.2, note that $a \notin \nu_t$. Observe that (N_e, ν, ρ_a, t) is not a star graph.

(ii) Now we provide Example 3.5 to show that if ν is not an equiprime fuzzy ideal of N then even if (N_e, ν, ρ_p, t) is a star graph with root vertex p we get $p \notin \nu_t$.

Example 3.5. Let N be the nearring with addition and multiplication defined as in Table 1. Define $\nu : N \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.8 & \text{if } x = 0, \\ 0.5 & \text{if } x = b, \\ 0.1 & \text{if } x \in \{a, c\}. \end{cases}$$

If thresholds are $\alpha = 0.5$ and $\beta = 0.8$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$ and the values of $\rho_b(b, x)$ are given in Table 4. Graph (N_e, ν, ρ_b, t) is shown in Figure 4.

	$x = 0$	$x = a$	$x = b$	$x = c$
$\rho_b(b, x)$	0.5	0.1	0	0.1

Table 4: $\rho_b(b, x)$ when $\nu_t = \{0\}$

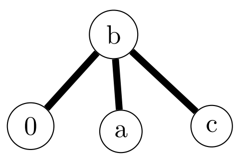


Figure 4: (N_e, ν, ρ_b, t) when $\nu_t = \{0\}$

Note that ν is not an equiprime fuzzy ideal of N $(\alpha \vee \mu(b) \vee \mu(b-0) = 0.5 \vee 0.5 \vee 0.5 = 0.5 \ngeq 0.8 = 0.8 \wedge 0.8 = \beta \wedge \inf_{r \in N} \mu(brb - br0))$ and (N_e, ν, ρ_b, t) is a star graph with root vertex b. Observe that $b \notin \nu_t$.

(iii) In Example 3.3, note that $7 \notin \nu_t$. Observe that 7 is not connected to all other vertices in (N_e, ν, ρ_e, t).

PROPOSITION 3.6. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$.*
(i) Let ν be an equiprime fuzzy ideal of N. Then ν_t is a strong vertex cut of (N_e, ν, ρ_e, t).

(ii) Let ν be an equisemiprime fuzzy ideal of N and ν_t be a strong vertex cut of (N_e, ν, ρ_e, t). Then ν is an equiprime fuzzy ideal of N.

Proof. To prove (i), let ν be an equiprime fuzzy ideal of N and $t \in (\alpha, \beta]$. Suppose $\nu_t = N$. Then ν_t is a strong vertex cut of (N_e, ν, ρ_e, t). Let $\nu_t \subset N$. Let $p \in N \setminus \nu_t$ and $x \in N \setminus \nu_t$ such that there exists an edge between p and x in (N_e, ν, ρ_e, t). Then $prx - pr0 \in \nu_t$ or $xrp - xr0 \in \nu_t$ for all $r \in N$. Without loss of generality assume $prx - pr0 \in \nu_t$ for all $r \in N$. As ν is an equiprime fuzzy ideal of N, we get ν_t is an equiprime ideal of N. Then $p \in \nu_t$ or $x \in \nu_t$. A contradiction to the fact that $p \in N \setminus \nu_t$ and $x \in N \setminus \nu_t$. Hence ν_t is a strong vertex cut of (N_e, ν, ρ_e, t).

To prove (ii), let ν be a equisemiprime fuzzy ideal of N and ν_t be a strong vertex cut of (N_e, ν, ρ, t). Let $prx - pro \in \nu_t$ for all $r \in N$. Suppose $p = x$. Then $p \in \nu_t$ (ν_t is equisemiprime.) Let $p \neq x$. Suppose $p \in N \setminus \nu_t$ and $x \in N \setminus \nu_t$. As ν_t is a strong vertex cut of (N_e, ν, ρ_e, t) there is no edge between p and x in (N_e, ν, ρ_e, t). Then $prx - pro \notin \nu_t$ and $xrp - xr0 \notin \nu_t$ for some $r \in N$. A contradiction since $prx - pr0 \in \nu_t$ for all $r \in N$. Hence ν_t is an equiprime ideal of N. Therefore ν is an equiprime fuzzy ideal of N. □

Now we provide examples to show the conditions in Proposition 3.6 are necessary.
(i) We provide Example 3.7 to show that if ν is not an equiprime fuzzy ideal of N. Then ν_t not a strong vertex cut of (N_e, ν, ρ_e, t).

(ii) We provide Example 3.8 to show that if ν is not an equisemiprime fuzzy ideal of N. Then even if ν_t is a strong vertex cut of (N_e, ν, ρ_e, t), ν is not an equiprime fuzzy ideal of N.

EXAMPLE 3.7. Let N be the nearring with addition $+$ and multiplication $\cdot$ defined as in Table 1. We define $\nu : N \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.8 & \text{if } x = 0, \\ 0.5 & \text{if } x = b, \\ 0.1 & \text{if } x \in \{a, c\}. \end{cases}$$

If we take thresholds $\alpha = 0.5$ and $\beta = 0.8$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$ and for $p \in N$ the values of $\rho_p(p, x)$ are given in Table 5. Graph (N_e, ν, ρ_e, t) is shown in Figure 5.

$\rho_p(p,x)$	$x=0$	$x=a$	$x=b$	$x=c$
$\rho_0(0,x)$	0	0.1	0.1	0.1
$\rho_a(a,x)$	0.1	0	0.5	0
$\rho_b(b,x)$	0.5	0.1	0	0.1
$\rho_c(c,x)$	0.1	0	0.1	0

Table 5: $\rho_0(x,y)$ when $\nu_t=\{0\}$

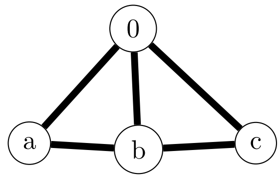


Figure 5: (N_e,ν,ρ_e,t) when $\nu_t=\{0\}$

Note that ν is not an equiprime fuzzy ideal of N ($\alpha\vee\nu(b)\vee\nu(b-0)=0.5\vee0.5\vee0.5=0.5\ngeq0.8=0.8\wedge0.8=\beta\wedge\inf_{r\in N}\nu(brb-br0)$.) Observe that $\nu_t=\{0\}$ is not a strong vertex cut of (N_e,ν,ρ_e,t).

Example 3.8. Let N=Z_4 be the ring of integers modulo 4. We define $\nu:N\to[0,1]$ by

$$\nu(x)=\begin{cases}0.9 & \text{if } x=0,\\ 0.6 & \text{if } x=2,\\ 0.3 & \text{if } x\in\{1,3\}.\end{cases}$$

Take thresholds $\alpha=0.6$ and $\beta=0.9$. Then ν is a fuzzy ideal of N. Let $t=\beta$. Then $\nu_t=\{0\}$ and the values of $\rho_p(p,x)$ are as in Table 6. The graph is given in Figure 6.

$\rho_p(p,x)$	$x=0$	$x=1$	$x=2$	$x=3$
$\rho_0(0,x)$	0	0.3	0.6	0.3
$\rho_1(1,x)$	0.3	0	0	0
$\rho_2(2,x)$	0.6	0	0	0
$\rho_3(3,x)$	0.3	0	0	0

Table 6: Values of $\rho_p(p,x)$ when $\nu_t=\{0\}$

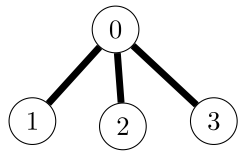


Figure 6: (N_e,ν,ρ_e,t) when $\nu_t=\{0\}$

Note that ν is not a equisemiprime fuzzy ideal of N since $\alpha\vee\nu(2)\vee\nu(2)=0.6\vee0.6\vee0.6=0.6\ngtr0.9=0.9\wedge0.9=\beta\wedge\inf_{r\in N}\nu(2r2-2r0)$ and $\nu_t=\{0\}$ is a strong vertex cut of (N_e,ν,ρ_e,t). Observe that ν is not an equiprime fuzzy ideal of N.

Proposition 3.9. *Let ν be an equiprime fuzzy ideal of N and $t\in(\alpha,\beta]$.*
(i) Let x be a vertex in (N_e,ν,ρ_e,t). If $\rho_p(p,x)>0\;\forall\,p\neq x;p\in N$ then $p\in\nu_t$.

(ii) Let x be a vertex in (N_e,ν,ρ_e,t). If $p\in\nu_t$ then $\rho_p(p,x)>0\;\forall\,p\neq x;p\in N$.

(iii) ν is an equiprime fuzzy ideal of N if and only if every element $x\in\nu_t$ is connected to all other elements of N in (N,ν,ρ_e,t).

(iv) If ν is an equiprime fuzzy ideal N then (ν_t,ν,ρ,t) is a complete subgraph of (N,ν,ρ_e,t).

Remark 3.10. (i) In Example 3.2, note that ν is an equiprime fuzzy ideal of N and $\rho_a(a,c)=0$ for $x=c$. Observe that $a\notin\nu_t$.

(ii) In Example 3.2, note that $c\notin\nu_t$. Observe that $\rho_c(c,a)=0$.

DEFINITION 3.11. Let (N_e, ν, ρ_e, t) be an equiprime fuzzy graph. Then (N_e, ν, ρ_e, t) is said to be *ideal symmetric* if for every pair of vertices a, b in (N_e, ν, ρ_e, t) with an edge between them, either $[\rho(a, c) > 0 \; \forall \; c \neq a; c \in N]$ or $[\rho(b, c) > 0 \; \forall \; c \neq b; c \in N]$.

PROPOSITION 3.12. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$.*
(a) If ν is an equiprime fuzzy ideal of N then (N_e, ν, ρ_e, t) is ideal symmetric.

(b) Suppose (i) (N, ν, ρ_e, t) is ideal symmetric; (ii) ν is equisemiprime fuzzy ideal of N; (iii) for every $x \in N$ and $\rho(x, z) > 0 \; \forall \; z \neq x; z \in N$ implies $x \in \nu_t$. Then ν is an equiprime fuzzy ideal of N.

Proof. Let $t \in (\alpha, \beta]$. To prove (a) let ν be an equiprime fuzzy ideal of N and $p, x \in N$ be such that there is an edge between p and x in (N_e, ν, ρ_e, t). Then for all $r \in N$ we get $prx - pr0 \in \nu_t$ or $xrp - xr0 \in \nu_t$. Without loss of generality, assume $prx - pr0 \in \nu_t$. Then $p \in \nu_t$ or $x \in \nu_t$ (as ν is an equiprime fuzzy ideal of N then ν_t is an equiprime ideal of N). By Proposition 3.9 (ii) we get $\rho(p, z) > 0 \; \forall \; z \neq p; z \in N$ or $\rho(x, z) > 0 \; \forall \; z \neq x; z \in N$. Hence (N, ν, ρ, t) is ideal symmetric.

To prove (b), let $p, x \in N$ and $prx - pr0 \in \nu_t$ for all $r \in R$. Suppose $\nu_t = N$. Then $p \in \nu_t$. Let $\nu_t \subset N$. Suppose $p = x$. Then $p \in \nu_t$ (by (ii) ν is an equisemiprime fuzzy ideal of N and ν_t is an equisemiprime ideal of N). Let $p \neq x$. Now there exists an edge between p and x in (N_e, ν, ρ, t). As (N_e, ν, ρ, t) is ideal symmetric we get either $\rho(p, z) > 0 \; \forall \; z \neq p; z \in N$ or $\rho(x, z) > 0 \; \forall \; z \neq x; z \in N$. By Proposition 3.9 (i), we get $p \in \nu_t$ or $x \in \nu_t$. Thus ν is an equiprime fuzzy ideal of N. □

If ν is not an equirprime fuzzy ideal of N then (N_e, ν, ρ_e, t) is not ideal symmetric. We provide Example 3.13.

EXAMPLE 3.13. Let $N = Z_6$ be the ring of integers modulo 6. We define $\nu : N \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.9 & \text{if } x = 0, \\ 0.7 & \text{if } x \in \{2, 4\}, \\ 0.3 & \text{if } x \in \{1, 3, 5\}. \end{cases}$$

If we take thresholds $\alpha = 0.7$ and $\beta = 0.9$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$ and the values of $\rho_p(p, x)$ are given in Table 7. The graph (N_e, ν, ρ_e, t) is shown in Figure 7.

$\rho_p(p, x)$	$x = 0$	$x = 1$	$x = 2$	$x = 3$	$x = 4$	$x = 5$
$\rho_0(0, x)$	0	0.5	0.7	0.5	0.7	0.5
$\rho_1(1, x)$	0.5	0	0	0	0	0
$\rho_2(2, x)$	0.7	0	0	0.5	0	0
$\rho_3(3, x)$	0.5	0	0.5	0	0.5	0
$\rho_4(4, x)$	0.7	0	0	0.5	0	0
$\rho_5(5, x)$	0.5	0	0	0	0	0

Table 7: $\rho_p(p, x)$ when $\nu_t = \{0\}$

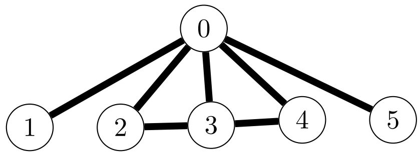


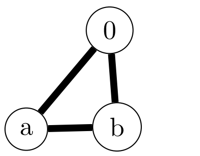
Figure 7: (N_e, ν, ρ_e, t) when $\nu_t = \{0\}$

Note that ν is not an equirpeime fuzzy ideal of N. $(\alpha \vee \nu(2) \vee \nu(3) = 0.7 \vee 0.7 \vee 0.3 = 0.7 \not\geq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \inf_{r \in N} \nu(2r3 - 2r0))$. Observe that the graph (N_e, ν, ρ_e, t) in Figure 7 is not ideal symmetric.

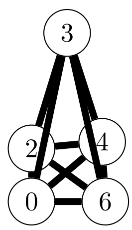
In Example 3.8, note that the graph in Figure 6 is ideal symmetric, however ν is not an equisemiprime fuzzy ideal of N. Since $\alpha \vee \nu(2) \vee \nu(2) = 0.6 \vee 0.6 \vee 0.6 = 0.6 \not\geq 0.9 = 0.9 \wedge 0.9 = \beta \wedge \inf_{r \in N} \nu(2r2 - 2r0)$. Observe that ν is not an equiprime fuzzy ideal of N.

Proposition 3.14. *Let ν be an equiprime fuzzy ideal of N, $t \in (\alpha, \beta]$ and $n \notin \nu_t$. Then $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N_e, ν, ρ_e, t).*

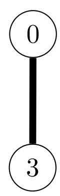
Proof. Let $t \in (\alpha, \beta]$. Then by Proposition 3.9 (iv), we get (ν_t, ν, ρ, t) is complete subgraph of (N_e, ν, ρ_e, t). Let $n \notin \nu_t$ and $C = \nu_t \cup \{n\}$. By Proposition 3.9 (iii), we get that every element of ν_t is connected to n. This proves that (C, ν, ρ_e, t) is complete. It remains to prove that C is maximal. First, we show that if $p \in N \setminus \nu_t, x \in N \setminus \nu_t$ and $p \neq x$ then p is not connected to x in (N_e, ν, ρ_e, t). If possible, let p be connected to x. Then for all $r \in N$ we get $prx - pr0 \in \nu_t$ or $xrp - xr0 \in \nu_t$. Without loss of generality, assume $prx - pr0 \in \nu_t$. As ν_t is an equiprime ideal of N, we get $p \in \nu_t$ or $x \in \nu_t$. This is a contradiction to the fact that $p \in N \setminus \nu_t, x \in N \setminus \nu_t$. Now, let $C_1 = C \cup \{m\}$; $m \notin \nu_t$ and $m \neq n$. Then $\rho_e(n, m) = 0 \neq \nu(m) \wedge \nu(n)$. This implies (C_1, ν, ρ_e, t) is not complete. Hence (C_1, ν, ρ_e, t) cannot be a fuzzy clique. Thus $C = \nu_t \cup \{n\}$ is a fuzzy clique of (N_e, ν, ρ_e, t). □



(a) Fuzzy clique of (N_e, ν, ρ_e, t) when $\nu_t = \{0, b\}$



(b) Fuzzy clique of (N_e, ν, ρ_e, t) when $\nu_t = \{0, 2, 4, 6\}$



(c) Complete subgraph of (N_e, ν, ρ_e, t) when $\nu_t = \{0\}$

Figure 8: Complete subgraphs of (N_e, ν, ρ_e, t)

Remark 3.15. (i) In Example 3.2, take $b \notin \nu_t$. Then observe that $C = \nu_t \cup \{b\}$ is a fuzzy clique of (N_e, ν, ρ_e, t) as shown in Figure 8a.

(ii) In Example 3.3, take $3 \notin \nu_t$. Then observe that $C = \nu_t \cup \{3\}$ is a fuzzy clique of (N_e, ν, ρ_e, t) as shown in Figure 8b.

(iii) In Example 3.13, note that ν is not an equiprime fuzzy ideal of N. If we take $3 \notin \nu_t$ then observe that $C = \nu_t \cup \{3\}$ is not a fuzzy clique of (N_e, ν, ρ_e, t) as shown in Figure 8c.

Proposition 3.16. *Let ν be an equiprime fuzzy ideal of N and $t \in (\alpha, \beta]$. Then*
(i) ν_t is a vertex cover of (N_e, ν, ρ_e, t). (ii) $(N_e \setminus \nu_t, \nu, \rho_e, t)$ is an empty graph.

Remark 3.17. (i) In Example 3.2, note that ν is an equiprime fuzzy ideal of N. Observe that ν_t is a vertex cover of (N_e, ν, ρ_e, t).

(ii) In Example 3.13, note that ν is not an equiprime fuzzy ideal of N. Note that $\nu_t = \{0\}$ is not a vertex cover of (N_e, ν, ρ_e, t).

(iii) In Example 3.13, note that ν is not an equiprime fuzzy ideal of N. Note that $(N_e \setminus \nu_t, \nu, \rho, t)$ is not an empty graph. In fact $(N_e \setminus \nu_t, \nu, \rho_e, t)$ is as shown in Figure 9.

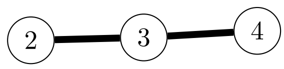


Figure 9: (N_e, ν, ρ_e, t) when $\nu_t = \{0\}$

Proposition 3.18. *Let N be a simple nearring and ν be an equiprime fuzzy ideal of N. Then (N_e, ν, ρ_e, t) is a star graph or (N_e, ν, ρ_e, t) is a complete graph.*

Remark 3.19. In Example 3.2, note that ν is an equiprime fuzzy ideal, however the nearring is not a simple nearring. Observe that the graph is neither a star graph nor a complete graph.

4. Interrrelations between fuzzy graphs

Proposition 4.1. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$. Then $(N_3, \nu, \rho_3, t) = (N_e, \nu, \rho_e, t)$ if any one of the following conditions are satisfied.*
(i) N is zero symmetric. *(ii) N is distributive.*
(iii) Every ideal of N is totally reflexive. *(iv) N is equiprime.*

Proof. Let $t \in (\alpha, \beta]$. Then ν_t is an ideal of N for all $t \in (\alpha, \beta]$. We have vertex set of $(N_3, \nu, \rho_3, t) = (N_e, \nu, \rho_e, t) = N$. To prove (i), suppose $p, x \in N$ such that there is an edge between p and x in (N_3, ν, ρ_3, t). Then $pNx \subseteq \nu_t$ or $xNp \subseteq \nu_t$. Without loss of generality assume $pNx \subseteq \nu_t$. Then $pnx \in \nu_t$ for all $n \in N$. Let $n \in N$ is arbitrarily fixed. Then we get $pnx - pn0 \in \nu_t$. (as N is zero symmetric $xn0 = 0$) Then p and x are adjacent in (N_e, ν, ρ_e, t). Therefore $E((N_3, \nu, \rho_3, t)) \subseteq E((N_e, \nu, \rho_e, t))$. Now suppose $p, x \in N$ such that (p, x) is an edge in (N_e, ν, ρ_e, t). Then $prx - pr0 \in \nu_t$ for all $r \in N$ or $xnp - xn0 \in \nu_t$ for all $r \in N$. Without loss of generality assume $prx - pr0 \in \nu_t$ for all $r \in N$. Let $r \in N$ be arbitrarily fixed. Then we get $prx \in N$ (as N is zero symmetric we get $pr0 = 0$). Hence we get $prx \in \nu_t$ for all $r \in N$. Then $pNx \subseteq \nu_t$, which implies p and x are adjacent in (N_3, ν, ρ_3, t). Hence $E((N_e, \nu, \rho_e, t)) \subseteq E((N_3, \nu, \rho_3, t))$. Therefore $E((N_e, \nu, \rho_e, t)) = E((N_3, \nu, \rho_3, t))$. Thus $(N_3, \nu, \rho, t) = (N_e, \nu, \rho_e, t)$.

The proofs of (ii), (iii) and (iv) are similar to that of (i). □

We provide examples to show that if conditions given in Proposition 4.1 are not satisfied then $(N_3, \nu, \rho_3, t) \neq (N_e, \nu, \rho_e, t)$.

Example 4.2. Let $N = \{0, a, b, c\}$ be the nearring with addition and multiplication defined as in Table 1. We define $\nu : N \to (0, 1]$ by

$$\nu(x) = \begin{cases} 0.9 & \text{if } x = 0, \\ 0.4 & \text{if } x = b, \\ 0.2 & \text{if } x \in \{a, c\}. \end{cases}$$

If we take thresholds $\alpha = 0.4$ and $\beta = 0.9$ then ν is a fuzzy ideal of N. Let $t = \beta$. Then $\nu_t = \{0\}$ and the values of $\rho_p(p, x)$ are as in Table 8 and the graph (N_e, ν, ρ_e, t) is given in Figure 10.

$\rho_p(p,x)$	$x=0$	$x=a$	$x=b$	$x=c$
$\rho_0(0,x)$	0	0.2	0.4	0.1
$\rho_a(a,x)$	0.2	0	0.4	0
$\rho_b(b,x)$	0.4	0.2	0	0.2
$\rho_c(c,x)$	0.2	0	0.2	0

Table 8: Values of $\rho_p(p, x)$ when $\nu_t = \{0\}$

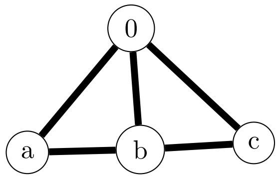


Figure 10: (N_e, ν, ρ, t) for $\nu_t = \{0\}$

The values of $\rho(x, y)$ are as in Table 9 and the graph (N_3, ν, ρ_3, t) is given in Figure 11.

$\rho(x,y)$	$y=0$	$y=a$	$y=b$	$y=c$
$x=0$	0	0.2	0.4	0.1
$x=a$	0.2	0	0.4	0
$x=b$	0.4	0.4	0	0
$x=c$	0.2	0	0	0

Table 9: Values of $\rho(x, y)$ when $\nu_t = \{0\}$

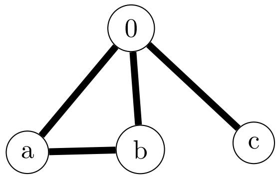


Figure 11: (N_3, ν, ρ_3, t) for $\nu_t = \{0\}$

In this example, note that:

(i) The nearring N is not zero symmetric $(b \cdot 0 \neq 0, c \cdot 0 \neq 0)$. Observe that $(N_3, \nu, \rho, t) \neq (N_e, \nu, \rho, t)$.

(ii) The nearring N is not distributive $(b(a + c) = b \cdot b = b$ and $b \cdot a + b \cdot c = b + b = 0$. Hence $b(a + c) \neq b \cdot a + b \cdot c)$. Observe that $(N_3, \nu, \rho, t) \neq (N_e, \nu, \rho, t)$.

(iii) The ideal ν_t is not totally reflexive $(aNb = \{0\} \in \nu_t$ however $bNa = \{b\} \notin \nu_t)$. Observe that $(N_3, \nu, \rho_3, t) \neq (N_e, \nu, \rho_e, t)$.

(iv) $\{0\}$ is not an equiprime ideal of N. Hence N is not an equiprime nearring. Observe that $(N_3, \nu, \rho_3, t) \neq (N_e, \nu, \rho_e, t)$.

PROPOSITION 4.3. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$.*
(i) If ν is a 3-prime fuzzy ideal of N then $(N_3, \nu, \rho_3, t) \subseteq (N_c, \nu, \rho_c, t)$.

(ii) If ν is a c-prime fuzzy ideal of N then $(N_3, \nu, \rho_3, t) = (N_c, \nu, \rho_c, t)$.

Proof. We have $V((N_3, \nu, \rho, t)) = V((N_c, \nu, \rho, t)) = N$. To prove (i), let ν be a 3-prime fuzzy ideal of N. Let $x, y \in N$ such that $(x, y) \in E((N_3, \nu, \rho_3, t))$. Then $xNy \subseteq \nu_t$ or $yNx \subseteq \nu_t$. Without loss of generality assume $xNy \subseteq \nu_t$. As ν is a 3-prime fuzzy ideal of N we get ν_t is a 3-prime ideal of N. Then we get $x \in \nu_t$ or $y \in \nu_t$. Now, let us assume $x \in \nu_t$. Then $xy \in \nu_t$ for all $y \in N$. Then $(x, y) \in E((N_c, \nu, \rho_c, t))$. Proof is similar for $y \in \nu_t$. Hence $E((N_3, \nu, \rho_3, t)) \subseteq E((N_c, \nu, \rho_c, t))$. Therefore $(N_3, \nu, \rho_3, t) \subseteq (N_c, \nu, \rho_c, t)$.

To prove (ii), let ν be a c-prime fuzzy ideal of N. Let $a, b \in N$ such that $(a, b) \in E((N_c, \nu, \rho_c, t))$. Then $ab \in \nu_t$ or $ba \in \nu_t$. Without loss of generality assume $ab \in \nu_t$. As ν is a c-prime fuzzy ideal of N we get ν_t is a c-prime ideal of N. Then we get $x \in \nu_t$ or $y \in \nu_t$. Now, let us assume $x \in \nu_t$. Then by the property of ideal we get $xN \subseteq \nu_t$. Then $xNy \subseteq \nu_t$ for all $y \in N$. Then $(x, y) \in E((N_3, \nu, \rho_3, t))$. Proof is similar for $y \in \nu_t$. Hence $E((N_c, \nu, \rho_c, t)) \subseteq E((N_3, \nu, \rho_3, t))$. Therefore $(N_c, \nu, \rho_c, t) \subseteq (N_3, \nu, \rho_3, t)$. Every c-prime fuzzy ideal is a 3-prime fuzzy ideal. Then from (i) we get $E((N_3, \nu, \rho_3, t)) \subseteq E((N_c, \nu, \rho_c, t))$. Therefore $(N_3, \nu, \rho_3, t) = (N_c, \nu, \rho_c, t)$. □

PROPOSITION 4.4. *Let ν be a fuzzy ideal of N and $t \in (\alpha, \beta]$. Then $(N_3, \nu, \rho_3, t) = (N_c, \nu, \rho_c, t) = (N_e, \nu, \rho_e, t)$, if any one of the following conditions are satisfied.*
(i) ν is a c-prime fuzzy ideal of a zero symmetic nearring N.

(ii) ν is a prime fuzzy ideal of a commutative ring N.

5. Nearring homomorphism and graph homomorphism

PROPOSITION 5.1. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomorphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then f is an one to one and onto graph homomorphism from (N_{1e}, ν, ρ_e, t) to $(N_{2e}, f(\nu), f(\rho)_e, t)$.*

Proof. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then by [18, Proposition 3.25] we get $f(\nu)$ is a fuzzy ideal of N_2 with same thresholds as that of ν. Let $p, x \in N_1$ such that $p \neq x$ and (p, x) be an edge of (N_{1e}, ν, ρ_e, t). Then for all $r \in N_1$ we get $prx - pr0_1 \in \nu_t$ or $xrp - xr0_1 \in \nu_t$ where 0_1 is the additive identity of N_1. Without loss of generality assume $prx - pr0_1 \in \nu_t$. Let $r \in N_1$ be arbitrarily fixed. Then $f(prx - pr0_1) \in f(\nu_t) \subseteq f(\nu)_t$ (by [18, Remark 3.30]). Then $f(prx - pr0_1) \in f(\nu)_t$. As f is a nearring homomorphism we get $f(prx - pr0_1) = f(p)f(r)f(x) - f(p)f(r)f(0_1) = f(p)f(r)f(x) - f(p)f(r)0_2 \in f(\nu)_t$ where 0_2 is the additive identity of N_2. As f is one to one we get $f(p) \neq f(x)$. Also $f(N_1) = N_2$ (f is onto). Hence $(f(p), f(x))$ is an edge in $(N_{2e}, f(\nu), f(\rho)_e, t)$. Therefore f is a graph homomorphism from (N_{1e}, ν, ρ_e, t) to $(N_{2e}, f(\nu), f(\rho)_e, t)$. Let $p_1, p_2 \in N_1$ such that $p_1 \neq p_2$. Then $f(p_1) \neq f(p_2)$ (since f is one to one). Hence there is one to one correspondence between the vertex set of (N_{1e}, ν, ρ_e, t) and $(N_{2e}, f(\nu), f(\rho)_e, t)$. Let (p_1, x_1) and (p_2, x_2) be edges of (N_{1e}, ν, ρ_e, t) such that $(p_1, x_1) \neq (p_2, x_2)$. Then for all $r_1 \in N_1$ we get $(p_1 r_1 x_1 - p_1 r_1 0_1) \in \nu_t$ or $(x_1 r_1 p_1 - x_1 r_1 0_1) \in \nu_t$ and for all $r_2 \in N_1$ we get $(p_2 r_2 x_2 - p_2 r_2 0_1) \in \nu_t$ or $(x_2 r_2 p_2 - x_2 r_2 0_1) \in \nu_t$. Without loss of generality assume $(p_1 r_1 x_1 - p_1 r_1 0_1) \in \nu_t$ and $(p_2 r_2 x_2 - p_2 r_2 0_1) \in \nu_t$. Let $r_1, r_2 \in N_1$ be arbitrarily fixed. Then $f(p_1 r_1 x_1 - p_1 r_1 0_1) = f(p_1)f(r_1)f(x_1) - f(p_1)f(r_1)f(0_1) \in f(\nu)_t$ and $f(p_2 r_2 x_2 - p_2 r_2 0_1) = f(p_2)f(r_2)f(x_2) - f(p_2)f(r_2)f(0_2) \in f(\nu)_t$. As f is one to one we get $f(p_1)f(r_1)f(x_1) - f(p_1)f(r_1)f(0_1) \neq f(p_2)f(r_2)f(x_2) - f(p_2)f(r_2)f(0_2)$. Therefore there is one to one correspondence between the edge sets of (N_{1e}, ν, ρ_e, t) and $(N_{2e}, f(\nu), f(\rho)_e, t)$. □

EXAMPLE 5.2. Let Z_n be the ring of integers modulo n. let N be the nearring defined in Example 3.2. Let $N_1 = Z_1 \times N = \{(0,0), (0,a), (0,b), (0,c)\}$ and $N_2 = N$. Define $f : N_1 \to N_2$ by $f((x,y)) = y$. Then f is an one to one and onto nearring homomorphism. We define $\nu : N_1 \to (0,1]$ by

$$\nu(x) = \begin{cases} 0.9 & \text{if } x = (0,0), \\ 0.5 & \text{if } x = (0,b), \\ 0.1 & \text{if } x \in \{(0,a),(0,c)\}. \end{cases}$$

If we take thresholds α=0.1 and β=0.5 then ν is a fuzzy ideal of N_1. Let t=β. Then ν_t=$\{(0,0),(0,b)\}$ and the values of $\rho_p(p,x)$ are as in Table 10 and the graph is given in Figure 12.

$\rho_p(p,x)$	$x=(0,0)$	$x=(0,a)$	$x=(0,b)$	$x=(0,c)$
$\rho_{(0,0)}((0,0),x)$	0	0.1	0.5	0.1
$\rho_{(0,a)}((0,a),x)$	0.1	0	0.5	0
$\rho_{(0,b)}((0,b),x)$	0.5	0.1	0	0.1
$\rho_{(0,c)}((0,c),x)$	0.1	0	0.1	0

Table 10: Values of $\rho(x,y)$ when $\nu_t = \{(0,0),(0,b)\}$

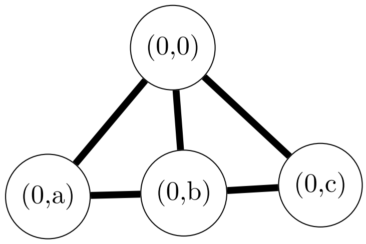


Figure 12: (N_e, ν, ρ_e, t) for $\nu_t = \{(0,0),(0,b)\}$

Then $f(\nu)$ is a fuzzy ideal of N_2 with the same thresholds as that of ν. Let $f(\nu) : N_2 \to (0,1]$ be given by

$$\nu(x) = \begin{cases} 0.7 & \text{if } x = 0, \\ 0.4 & \text{if } x = b, \\ 0.1 & \text{if } x \in \{a, c\}. \end{cases}$$

Then $f(\nu_t) = f(\{(0,0),(0,b)\}) = \{0, b\}$ is an ideal of N_2. The values of ρ are as in Table 11 and the graph is given in Figure 13.

$\rho_p(p,x)$	$x = 0$	$x = a$	$x = b$	$x = c$
$\rho_0(0,x)$	0	0.1	0.4	0.1
$\rho_a(a,x)$	0.1	0	0.4	0
$\rho_b(b,x)$	0.4	0.1	0	0.1
$\rho_c(c,x)$	0.1	0	0.1	0

Table 11: Values of $\rho_p(p,x)$ when $\nu_t = \{0, b\}$

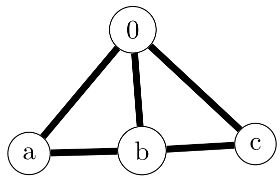


Figure 13: (N_e, ν, ρ_e, t) for $\nu_t = \{0, b\}$

Observe that f is an one to one graph homomorphism from N_1 to N_2.

PROPOSITION 5.3. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomorphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If $x \in \nu_t$ then $\rho(f(x), f(z)) > 0 \;\forall\; f(z) \neq f(x); f(z) \in N_2$.*

Proof. Let $t \in (\alpha, \beta]$ and $x \in \nu_t$.
Case 1. Suppose $p = 0_1$ where 0_1 is the additive identity of N_1. Then $f(p) = f(0_1) = 0_2$ where 0_2 is the additive identity of N_2. Then $(prx - pr0_1) = 0_1 \in \nu_t$ for all $x \in N_1$. Let $r \in N_1$ be arbitrarily fixed. As f is a nearring homomorphism we get $f(prx - pr0_1) = f(p)f(r)f(x) - f(p)f(r)f(0_1) = 0_2 f(r)f(x) - 0_2 f(r)0_2 = 0_2$, which is true for all $f(x) \in N_2$. Then $\rho(f(p), f(x)) > 0$ for all $f(p) \neq f(x)$.

Case 2. Let $p \neq 0_1$. As $p \in \nu_t$ and ν_t is an ideal, then for $r \in N_1$ we get $prx \in \nu_t$ and $pr0 \in \nu_t$ for all $x \in N_1$. Then $prx - pr0 \in \nu_t$. Then $f(prx - pr0) \in f(\nu_t) \subseteq f(\nu)_t$ for all $f(x) \in N_2$. As f is a nearring homomorphism we get $f(p)f(r)f(x) - f(p)f(r)f(0_1) \in f(\nu)_t$. As f is one to one we get $f(p) \neq f(x)$. Hence $(f(p), f(x)) \in E(((N_2)_e, f(\nu), f(\rho)_e, t))$. Therefore $\rho(f(p), f(x)) > 0 \;\forall\; f(p) \neq f(x); f(x) \in N_2$. □

REMARK 5.4. In Example 5.2, note that $(0, a) \notin \nu_t$. Then $f((0, a)) = a$ is not connected to all other vertices of N_2. Also $(0, b) \in \nu_t$. Then $f((0, b)) = b$ is connected to all other vertices of N_2.

PROPOSITION 5.5. *Let $f : N_1 \to N_2$ be an one to one and onto nearring homomorphism. Let ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If $\rho_{f(p)}(f(p), f(x)) > 0 \;\forall\; f(p) \neq f(x); f(x) \in N_2$ then $\rho_p(p, x) > 0 \;\forall\; p \neq x; x \in N_1$.*

REMARK 5.6. In Example 5.2, note that $c = f((0, c))$ is not connected to all other vertices of N_2. Observe that $(0, c)$ is not connected to all other vertices of N_1.

DEFINITION 5.7. Let $f : N_1 \to N_2$ be an onto nearring homomorphism, ν be a fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. Then f is said to preserve vertex covers of an equiprime fuzzy graph if for each ν_t vertex cover of (N_{1e}, ν, ρ_e, t), $f(\nu)_t$ is a vertex cover of $(N_{2e}, f(\nu), f(\rho)_e, t)$.

Remark 5.8. In Example 5.2, note that $\nu_t = \{(0,0),(0,b)\}$ is a vertex cover of (N_{1e}, ν, ρ_e, t) and $f(\nu)_t = \{0, b\}$ is a vertex cover of $(N_{2e}, f(\nu), f(\rho)_e, t)$.

Proposition 5.9. *Let $f : N_1 \to N_2$ be a nearring homomorphism, ν be an equiprime fuzzy ideal of N_1 and $t \in (\alpha, \beta]$. If (i) f preserves vertex covers, (ii) $f(\nu)_t$ is equisemiprime, then $f(\nu)$ is an equiprime fuzzy ideal of N_2.*

Proof. Let ν be an equiprime fuzzy ideal of N_1. Then $f(\nu)$ is a fuzzy ideal of N_2 with the same thresholds as that of ν. Suppose $\nu_t = N_1$. Then $f(\nu_t) = f(\nu)_t = N_2$ is an equiprime ideal of N_2. Then $f(\nu)$ is an equiprime fuzzy ideal of N_2. Now suppose $\nu_t \subset N_1$. As ν is an equiprime fuzzy ideal of N_1 we get ν_t is an equiprime ideal of N_1. By Proposition 3.16 (i) we get ν_t is a vertex cover of (N_{1e}, ν, ρ_e, t). As f preserves vertex cover we get $f(\nu)_t$ is a vertex covers of $(N_{2e}, f(\nu), f(\rho)_e, t)$. Let $(p, x) \in E(N_2, f(\nu), f(\rho)_e, t)$. Then $p \in f(\nu)_t$ or $x \in f(\nu)_t$. Then by Proposition 3.9, we get p is connected to all other vertices of $(N_{2e}, f(\nu), f(\rho)_e, t)$ or x is connected to all other vertices of $(N_{2e}, f(\nu), f(\rho)_e, t)$. Hence $(N_{2e}, f(\nu), f(\rho)_e, t)$ is ideal symmetric. Let $p \in N_2$ such that p is connected to all other vertices of $(N_{2e}, f(\nu), f(\rho)_e, t)$. Then for all $r \in N_2$ we get $prx - pr0 \in f(\nu)_t$ for all x in N_2. Suppose $p = x$. Then $p \in f(\nu)_t$ (since $f(\nu)_t$ is equisemiprime). As f is onto, $p = f(p_1)$ for some $p_1 \in N_1$. Now $f(p_1)$ is connected to all other vertices of $(N_{2e}, f(\nu), f(\rho), t)$. Then by Proposition 5.5, we get p_1 is connected to all other vertices of (N_{1e}, ν, ρ_e, t). Then for all $r_1 \in N_1$ we get $(p_1 r_1 x_1 - p_1 r_1 0_1) \in \nu_t$ for all $x_1 \in N_1$. Suppose $\nu_t = N_1$. Then $f(\nu)_t = N_2$ is ae equiprime ideal of N_2. Let $\nu_t \subset N_1$. Choose $x_1 \in N_1 \setminus \nu_t$, then $p_1 \in \nu_t$. (As ν is an equiprime fuzzy ideal of N_1, ν_t is an equiprime ideal of N_1.) Then $f(p_1) \in f(\nu)_t$ and $p \in f(\nu)_t$. Hence $(N_{2e}, f(\nu), f(\rho), t)$ satisfies all conditions of the Proposition 3.12. Therefor $f(\nu)$ is an equiprime fuzzy ideal of N_2. □

Remark 5.10. In Example 5.2, note that ν is an equiprime fuzzy ideal of N_1 and $f(\nu)$ is an equiprime fuzzy ideal of N_2.

Acknowledgement. We thank the anonymous reviewers and the editor for their constructive comments and suggestions for improving the paper.

The first author thanks St. Joseph Engineering College, Vamanjoor Mangaluru, for encouragement. The second and third authors thank Manipal Institute of Technologyand Manipal Academy of Higher Education for their encouragement.

References

[1] M. Akram, *m-polar fuzzy graphs*, Stud. Fuzziness Soft Comput., **371** (2019).

[2] M. Akram, *On T-fuzzy ideals in nearrings*, Int. J. Math. Math. Sci., **2007** (2007), Article ID 73514.

[3] M. Akram, W. A. Dudek, *Interval valued fuzzy graphs*, Comput. Math. Appl., **61** (2011), 289–299.

[4] M. Akram, S. Nawaz, *On soft fuzzy graphs*, Ital. J. Pure Appl. Math., **34** (2015), 497–514.

[5] F. W. Anderson, K. R. Fuller, *Rings and Categories of Modules*, Second edition, Springer-Verlag, USA, 1992.

[6] I. Beck, *Coloring of commutative rings*, J. Algebra, **116** (1998), 208–226.

[7] S. K. Bhakat, P. Das, *Fuzzy subrings and ideals redefined*, Fuzzy Sets Syst., **81** (1996), 383–393.

[8] S. Bhavanari, S. P. Kuncham, B. S. Kedukodi, *Graph of a nearring with respect to an ideal*, Commun. Algebra, **38** (2010), 1957–1962.

[9] B. Davvaz, *Fuzzy ideals of nearring with interval valued membership functions*, J. Sci. Islam. Repub. Iran, **12** (2001), 171–175.

[10] B. Davvaz, $(\epsilon, \epsilon \vee q)$*-fuzzy subnear-rings and ideals*, Soft Comput., **10** (2006), 206–211.

[11] J. A. Goguen, *L-fuzzy sets*, J. Math. Anal. Appl., (1967), 145–174.

[12] N. J. Groenewald, *Different prime ideals in near-rings*, Comm. Algebra, **19** (1991), 2667–2675.

[13] F. Harary, *Graph theory*, Naraosa Publishing House, India 2001.

[14] P. Hell, J. Nesetril, *Graphs and Homomorphisms*, Oxford University press New York 2004.

[15] B. Jagadeesha, B. S. Kedukodi, S. P. Kuncham, *Interval Valued L-fuzzy Ideals based on t-norms and t-conorms*, J. Intell. Fuzzy Systems, **28(6)** (2015), 2631–2641.

[16] B. S. Kedukodi, B. Jagadeesha, S. P. Kuncham, S. Juglal, *Different prime graphs of a nearring with respect to an ideal*, Nearrings, Nearfields and related topics, (2017) 185–203.

[17] B. S. Kedukodi, S. P. Kuncham, S. Bhavanari, *Nearring ideals, graphs and cliques*, Int. Math. Forum, (2013), 73–83.

[18] B. S. Kedukodi, S. P. Kuncham, S. Bhavanari, *Equiprime, 3-prime and c-prime fuzzy ideals of nearrings*, Soft Comput., **13** (2009), 933–944.

[19] B. S. Kedukodi, S. P. Kuncham, B. Jagadeesha, *Interval valued L-fuzzy prime ideals, triangular norms and partially ordered groups*, Soft Comput., **23** (2019), 907–920.

[20] L. T. Koczy, *Fuzzy graphs in the evaluation and optimization of networks*, Fuzzy Sets Syst., **26** (1992), 307–319.

[21] J. N. Mordeson, P. S. Nair, *Fuzzy Graphs and Fuzzy Hypergraphs*, Physica-Verlag, Heidelberg, 2000.

[22] J. N. Mordeson, C. Peng, *Operations on fuzzy graphs*, Inform Sci., **79** (1994), 159–170.

[23] C. Pang, R. Zhang, Q. Zhang, J. Wang, *Dominating sets in directed graphs*, Inform Sci., **180** (2010), 3647–3652.

[24] G. Pilz, *Near-Rings*, Revised edition, North Holland, 1983.

[25] S. P. Redmond, *An ideal-based zero-divisor graph of a commutative ring*, Comm. Algebra, **31** (2003), 4425–4443.

[26] K. P. Shum, M. Akram, *Intuitionistic fuzzy (T,S)-ideals in nearrings*, J. Algebra Discrete Struct., **6** (2008), 37–52.

[27] S. Veldsman, *On equiprime near-rings*, Comm. Algebra, **20(9)** (1992), 2569–2587.

(received 18.11.2021; in revised form 12.09.2022; available online 17.04.2023)

Department of Mathematics, St Joseph Engineering College, Vamanjoor Mangaluru 575028, Karnataka India

E-mail: jagadeeshab@sjec.ac.in

Department of Mathematics, Manipal Institute of Technology, MAHE, Manipal, Karnataka 576104, India

E-mail: babushrisrinivas.k@manipal.edu

Department of Mathematics, Manipal Institute of Technology, MAHE, Manipal, Karnataka 576104, India

E-mail: syamprasad.k@manipal.edu